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Equilibria in repeated games of incomplete information: The general symmetric case

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Abstract. Every two person repeated game of symmetric incomplete information, in which the signals sent at each stage to both players are identical and generated by a state and moves dependent probability distribution on a given finite alphabet, has an equilibrium payoff.

Key words: Repeated games of incomplete information, stochastic games

1. Introduction

This paper proves the existence of equilibrium payoffs for incomplete information repeated two person games with symmetric random signals.

The first study, in the deterministic zero sum case, is due to Kohlberg and Zamir (1974). They show the existence of a value by reducing the problem to the study of stochastic games with absorbing states (Kohlberg, 1974). This result was then extended in two directions: by Forges (1982) to the zero-sum random signal case and by Neyman and Sorin (1997) to the deterministic non zero sum case.

The framework is given by a finite set K of states and for each state k in K, by a bi-matrix game G^k defined by $I \times J$ real valued payoff matrices A^k, B^k and $I \times J$ "signalling matrices" H^k with values in the set $\Delta(H)$ of probabilities on a finite set H.

For any initial distribution p on K, the game $\Gamma(p)$ is played as follows. First, the state k in K is chosen once for all according to p. The value of k is not announced to the players. Then there is infinite number of stages where at stage n, player I (resp. player II) chooses $i_n \in I$ (resp. $j_n \in J$). The payoff at that stage is thus $(a^k_{i_n,j_n}, b^k_{i_n,j_n})$ (for player I and II respectively), but is not announced. Rather the players are told a "public signal" h_n whose conditional distribution given the past is $H^k_{i_n,j_n}$. For the signal to contain all the information of the players at that stage and for perfect recall to hold, the signal reveals the moves: $i \neq i'$ or $j \neq j'$ implies that the distributions of $H^k_{i,j}$ and $H^{k'}_{i',j'}$ have disjoint supports.

2. The result

Any pair of strategies σ of player 1 and τ of player 2, together with the initial probability p, defines a probability distribution $P_{p,\sigma,\tau}$ on plays $(k, i_1, j_1, h_1, \ldots, i_n, j_n, h_n, \ldots)$ and therefore it also induces a probability distribution on the stream of payoffs $(x_1, y_1), \ldots, (x_n, y_n), \ldots,$ where $(x_t, y_t) = (a^k_{i_t,j_t}, b^k_{i_t,j_t})$. Let $x_t(\sigma, \tau) = E_{p,\sigma,\tau}(a^k_{i_t,j_t})$ be the expected payoff of player 1 at stage t, and set $\bar{x}_n(\sigma, \tau) = (1/n)\sum_{t=1}^{n} x_t(\sigma, \tau)$ to be the average expected payoff of player 1 up to stage n and similarly for player 2.

A history of length m is a sequence $\omega_m = (i_1, j_1, h_1, \ldots, i_m, j_m, h_m)$. Such histories generate an algebra $\mathscr{F}_m$ on the set $K \times (I \times J \times H)^{\infty}$. p_{m+1} is the conditional distribution on K given $\mathscr{F}_m$ induced by $P_{p,\sigma,\tau}$. Therefore any pair of strategies defines a martingale p_m, $m = 1, 2, \ldots$ (with $p_1 = p$), which reflects the information (equivalently the uncertainty) that the players have at each stage m about the state k in K.

A payoff vector $(a, b) \in \mathbb{R}^2$ is an ε-equilibrium payoff if there exist strategies σ of player 1 and τ of player 2 and a positive integer $N = N(\varepsilon)$ such that for any pair of strategies, σ' of player 1 and τ' of player 2, and any $n \geq N$,

$$\bar{x}_n(\sigma, \tau) + \varepsilon > a > \bar{x}_n(\sigma', \tau) - \varepsilon \quad (1)$$

and

$$\bar{y}_n(\sigma, \tau) + \varepsilon > b > \bar{y}_n(\sigma, \tau') - \varepsilon \quad (2)$$

(see Mertens, Sorin and Zamir (1994), p. 403).

Such a pair of strategies, σ of player 1 and τ of player 2, is called an ε-*uniform equilibrium* with payoff (a, b). An alternative equivalent property is that there exist $N = N(\varepsilon)$, such that for all $n, m \geq N$ and every strategy pair, σ' of player 1 and τ' of player 2,

$$\bar{x}_n(\sigma, \tau) > \bar{x}_m(\sigma', \tau) - \varepsilon$$

and

$$\bar{y}_n(\sigma, \tau) > \bar{y}_m(\sigma, \tau') - \varepsilon.$$

The above definition implies that any ε-uniform equilibrium with payoff (a, b) induces in fact an ε-equilibrium with payoff within ε of (a, b) in any sufficiently long game, or in any game with large enough discount factor.

E_ε denotes the set of all ε-uniform equilibrium payoff vectors in $\Gamma(p)$. The set of equilibrium payoffs in $\Gamma(p)$, $E_0(p)$, is defined as $\bigcap_{\varepsilon>0} E_\varepsilon(p)$. Note that

$E_0(p)$ is not empty if and only if, for every $\varepsilon > 0$, there exists an ε-uniform equilibrium.

Theorem. *For any two person repeated game with symmetric information $\Gamma(p)$, $E_0(p)$ is non empty.*

3. Examples

We first illustrate by two examples the way information propagates and then give some hints of the proof.

The first example deals with a zero sum game and is taken from Mertens (1982). The state space is $K = \{L, M, R\}$ and the initial probability p on K is uniform. The payoffs are given by

0	0
0	4

L

0	−2
2	0

M

0	0
−4	−2

R

and the signals by

ℓm	ℓ
p	q

L

ℓm	mr
p	q

M

r	mr
p	q

R

The value of each matrix is obviously 0. Moreover if (*Top, Left*) is played, ℓm will occur with probability 2/3 and r with probability 1/3. In this second case, the game R is revealed and one can assume that the payoff from then on is 0. Otherwise the game from this stage on is LM, with initial prior (1/2,1/2,0); the moves are non revealing except (*Top, Right*) which is completely revealing and thus leads to the payoff 0. Hence the analysis of game LM reduces to the analysis of the following

0	0*
1	2

where a star * denotes an absorbing payoff. This stochastic game has value 1. A similar analysis applies if (*Top, Left*) is played. Finally if player 1 plays *Bottom*, there is no change in information on K and the payoff is the expectation. The initial game thus is asymptotically equivalent to the following

$((2/3) \times 1 + (1/3) \times 0)^*$	$((2/3) \times (-1) + (1/3) \times 0)^*$
$-1/3$	$1/3$

which is again a stochastic game with absorbing states, hence has a value (Kohlberg, 1974). A similar reduction applies to any game in the zero-sum deterministic case (Kohlberg and Zamir, 1974). Note that the conditional probability on the state space can take only a finite number of values and when its value changes its support decreases. Thus an induction analysis based on the size of K is available. In the non zero sum deterministic case a similar procedure is feasible (Neyman and Sorin, 1997), replacing the value by an equilibrium in the reduced game with absorbing states (Vrieze and Thuijsman, 1989).

The second example is a one person decision problem where the decision maker is uninformed, in the spirit of a "bandit problem", with payoffs given by

L	M
-10	-10
4	0
0	4

and signals satisfying

L	M
$(2/3)a + (1/3)a'$	$(1/3)a + (2/3)a'$
b	b
c	c

Assume a uniform initial probability. The player will first play *Top* during a large number of periods then optimally in the revealed game. In fact the martingale of posteriors given *Top*, ($p_a = (2/3, 1/3)$, $p_{aa'} = (1/2, 1/2)$, $p_{aa} = (4/5, 1/5), \ldots$) will converge, hence in this case reach the boundary with probability one. Here again the natural state space is the set of posterior probabilities but it is unbounded. Note that in this case the player has in fact an optimal strategy: play *Top* on an infinite set of stages with zero density and optimally in the one stage game given the statistical information otherwise, but recall that already in two person zero sum games with absorbing states, optimal strategies may not exist (Blackwell and Ferguson, 1968). When 2 players are present, they both control the martingale and a backwards analysis based on the limit points is impossible.

In the previous examples, ε-equilibrium strategies at stage m depend only on the posterior at that stage. In the general case the computation of ε-optimal strategies will take into account the current value of the martingale of posterior probabilities and the number of stages where this value has changed. In fact the finiteness assumption on I and J implies that for any positive ε and any strategy pair, there is finite a number of jumps, say M, after which, with probability greater than ε, the martingale will be within ε of the boundary, hence the possibility of an induction analysis.

Explicitly the strategies will be constructed as follows: at the M^{th} jump, choose in the boundary of $\Delta(K)$ a closest point p_* to the current value p of the martingale and play an equilibrium in $\Gamma(p_*)$ from this stage on. This defines payoffs $e(M,p)$. Inductively payoffs $e(m,p)$ are defined on $\Delta(K)$ after m jumps ($m \le M$). After $m-1$ jumps, the players play at p equilibrium strategies in the stochastic game where the payoff is the average if the posterior does not change and is, after a jump, absorbing and equals to $e(m,p')$ where p' is the current posterior.

Hence the state space will be a product $\Delta(K) \times \{1,2,\ldots,M\}$, like in the picture below.

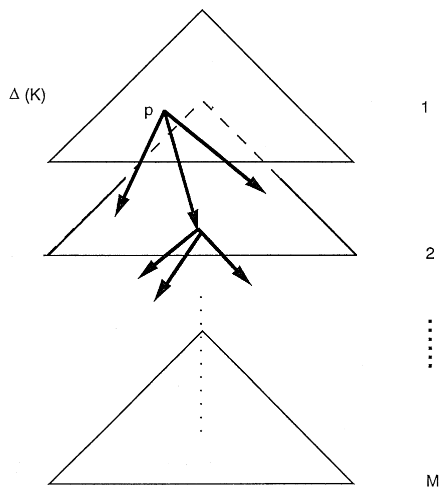

Fig. 1

4. The proof

a) Preliminaries

The proof is by induction on the number of elements in the support of p, hence we assume $E_0(p) \neq \emptyset$ for p in the boundary $b\Delta(K)$ of $\Delta(K)$.

We assume without loss of generality that all payoffs are bounded in absolute value by 1. Therefore $E_0(p) \neq \emptyset$ if and only if $E_0(p) \cap [-1,1] \times [-1,1] \neq \emptyset$. Note also that a Lipschitz property holds: the payoffs induced by a pair of strategies in $\Gamma(p)$ and $\Gamma(p')$ differ by at most $\|p-p'\|_1 = \sum_k |p^k - p'^k|$. In particular an ε-uniform equilibrium in $\Gamma(p)$ is an $(\varepsilon + \|p-p'\|_1)$-uniform equilibrium in $\Gamma(p')$.
Given $1/2 > \delta > 0$, let $\Delta_\delta(K) = \{p \in \Delta(K) | \forall k \in K, p^k \ge [p\delta/2K\}$. Then the non-emptiness of $E_0(p)$ for $p \in b\Delta(K)$ implies that $E_\delta(p) \cap [-1,1]^2$ is also non empty for $p \in \Delta \backslash \Delta_\delta(K)$.

b) The posterior distribution

Let $\tilde{q}(p,i,j)$ be the distribution of the posterior probability on $\Delta(K)$, when the prior is p and the moves played by the players are (i,j). Formally, define first a function $q : \Delta(K) \times I \times J \times H \to \Delta(K)$ satisfying

$$H^p_{i,j}(h) q^k(p,i,j,h) = p^k H^k_{i,j}(h)$$

where $H^p_{i,j}(h) = \sum_\ell p^\ell H^\ell_{i,j}(h)$. Now for each $(p,i,j) \in \Delta(K) \times I \times J$, $\tilde{q}(p,i,j)$ has the following distribution:

$$Prob(\tilde{q}(p,i,j) = q(p,i,j,h)) = H^p_{i,j}(h).$$

Let NR denote the subset of $I \times J$ for which $\tilde{q}(p,i,j)$ is the constant p, for all p. These are the *non revealing* entries where the signal h is non informative and the posterior does not change. The set of revealing entries, $(I \times J)\backslash NR$, is denoted by R.

From the definition of R and the fact that I and J are finite we deduce that $\exists \eta > 0$ such that $\forall p \in \Delta_\delta(K)$ and $\forall (i,j) \in R$,

$$E\left(\sum_k (\tilde{q}^k(p,i,j) - p^k)^2\right) > \eta. \tag{3}$$

c) The auxiliary games

We introduce now a new state space $\overline{K} = \Delta(K) \times \{0,1,\ldots,M\}$, where M is an integer to be fixed later and we define inductively mappings α, β from $\overline{K}$ to $[-1,1]$ as follows:

$(\sigma(M,p), \tau(M,p))$ are δ-uniform equilibrium strategies with payoffs $(\alpha(M,p), \beta(M,p))$ in the game $\Gamma(p)$ for $p \in \Delta\backslash\Delta_\delta(K)$ (which exist by the induction hypothesis on the number of elements in the support of p and the above remark). We write n_1 for the corresponding $N(\delta)$ (see (1), (2)). The strategy pair is arbitrarily defined for $p \in \Delta_\delta(K)$ and $(\alpha(M,p), \beta(M,p))$ are taken to be 0 there.

For $\ell = 0,1,\ldots,M-1$ and $p \in \Delta\backslash\Delta_\delta(K)$, let $(\alpha(\ell,p), \beta(\ell,p)) = (\alpha(M,p), \beta(M,p))$. Now for $\ell = 0,1,\ldots,M-1$ and $p \in \Delta_\delta(K)$ we define by backward procedure the game with absorbing payoffs $G(\ell,p)$ played on $I \times J$ and where the (i,j) entry is:

$$G(\ell,p)_{ij} = \begin{cases} \sum_k p^k (a^k_{i,j}, b^k_{i,j}) & \text{if } (i,j) \in NR, \\ \{E(\alpha(\ell+1, \tilde{q}(p,i,j)), \beta(\ell+1, \tilde{q}(p,i,j)))\}^* & \text{if } (i,j) \in R \end{cases}$$

where as usual a * denotes an absorbing payoff. By the theorem of Vrieze and Thuijsman (1989) (see also Mertens, Sorin and Zamir, 1994, p. 406–408) these games have ε_0-uniform equilibria strategies $(\sigma(\ell,p), \tau(\ell,p))$ with payoffs $(\alpha(\ell,p), \beta(\ell,p))$. Moreover, the Lipschitz property allows to choose the ε_0-uniform equilibria strategies $(\sigma(\ell,p), \tau(\ell,p))$ so that the positive integers $N(\varepsilon_0, \ell, p)$ associated with them (see (1), (2)) are independent of p and ℓ, and we thus denote $n_2 = N(\varepsilon_0, \ell, p)$.

d) The equilibrium strategies

On the space of plays we define W_ℓ to be the stopping time corresponding to the ℓ-th time a revealing entry is played, $\ell = 1, \dots, M$ and θ to be the entrance time in $\Delta(K)\backslash\Delta_\delta(K)$. Let $T_\ell = \min(W_\ell, \theta)$.

We now construct a pair of strategies (σ^*, τ^*) in $\Gamma(p)$ as follows:
(σ^*, τ^*) coincides with $(\sigma(0,p), \tau(0,p))$ until time T_1. Note that the hypothesis on the support of the signals implies that standard signalling holds in $\Gamma(p)$ and thus the strategies are well defined.
Then, inductively given the past history $\omega_{T_\ell} = (i_1, j_1, h_1, \dots, i_{T_\ell}, j_{T_\ell}, h_{T_\ell})$, (σ^*, τ^*) follows $(\sigma(\ell, p(\ell)), \tau(\ell, p(\ell)))$, from time $T_\ell + 1$ until time $T_{\ell+1}$, $\ell = 1, \dots, M$, where $p(\ell)$ is the posterior distribution on K given the past history ω_{T_ℓ}. More precisely for every history ω, $(\sigma^*, \tau^*)(\omega_{T_\ell}, \omega) = (\sigma(\ell, p(\ell)), \tau(\ell, p(\ell)))(\omega)$.

e) The payoffs

Fix a positive integer n which is greater than $n_0 = \max(n_1, n_2)$, and define the stopping times $S_\ell = \min(T_\ell, n), \ell = 1, \dots, M,\ S_0 = 0$. Set $a(\ell) = \alpha(\ell, p(\ell))$ and let $\mathscr{H}_\ell$ be the algebra of histories up to stage S_ℓ. The definition of σ^* and τ^* implies that for every $\ell = 0, \dots, M-1$, and for every strategy σ of player 1,

$$E_{\sigma^*,\tau^*}\left(\sum_{t=S_\ell+1}^{S_{\ell+1}} x_t + (n - S_{\ell+1})a(\ell+1)|\mathscr{H}_\ell\right) \geq (n - S_\ell)(a(\ell) - \varepsilon_0) - n_0 \quad (4)$$

and

$$E_{\sigma,\tau^*}\left(\sum_{t=S_\ell+1}^{S_{\ell+1}} x_t + (n - S_{\ell+1})a(\ell+1)|\mathscr{H}_\ell\right) \leq (n - S_\ell)(a(\ell) + \varepsilon_0) + n_0. \quad (5)$$

Also, on $p(M) \notin \Delta_\delta(K)$

$$E_{\sigma^*,\tau^*}\left(\sum_{t=S_M+1}^{n} x_t|\mathscr{H}_M\right) \geq (n - S_M)(a(M) - \delta) - n_0 \quad (6)$$

and

$$E_{\sigma,\tau^*}\left(\sum_{t=S_M+1}^{n} x_t|\mathscr{H}_M\right) \leq (n - S_M)(a(M) + \delta) + n_0 \quad (7)$$

for every strategy σ of player 1. Remark that

$$\left|\sum_{t=S_M+1}^{n} x_t - \sum_{t=S_M+1}^{n} x_t I(p(M) \notin \Delta_\delta(K))\right| \leq (n - S_M)I(p(M) \in \Delta_\delta(K)). \quad (8)$$

Note also that the event $\{p(M) \in \Delta_\delta(K)\}$ is included in the event $\{T_M < \theta\}$. Taking expectation in inequalities (4), (5), and summing the resulting equations over $\ell = 0, \dots, M-1$ we deduce that

$$E_{\sigma^*,\tau^*}\left(\sum_{t=1}^{S_M} x_t + (n - S_M)a(M)\right) \geq na(0) - \varepsilon_0 nM - n_0 M$$

and

$$E_{\sigma,\tau^*}\left(\sum_{t=1}^{S_M} x_t + (n - S_M)a(M)\right) \leq na(0) - \varepsilon_0 nM + n_0 M.$$

Adding to the above two inequalities the expectation of inequalities (6) and (7) respectively and using (8) we conclude that

$$E_{\sigma^*,\tau^*}\left(\sum_{t=1}^{n} x_t\right) \geq na(0) - \varepsilon_0 nM - n_0(M+1) - \delta n - nE_{\sigma^*,\tau^*}(I(T_M < \theta)) \tag{9}$$

and

$$E_{\sigma,\tau_*}\left(\sum_{t=1}^{n} x_t\right) \leq na(0) + \varepsilon_0 nM + n_0(M+1) + \delta n + nE_{\sigma,\tau^*}(I(T_M < \theta)) \tag{10}$$

f) The bound on M

Recall that p_{m+1} denotes the posterior probability on $\Delta(K)$ given $\mathscr{F}_m$, the algebra generated by the histories $\omega_m = (i_1, j_1, \dots, i_m, j_m, h_m)$. Let $v_m = (i_1, j_1, \dots, i_m, j_m)$ and denote by $\mathscr{G}_m$ the corresponding σ-algebra. From (3) it follows that $\exists \eta > 0$ such that for every strategy pair σ, τ:

$$E_{\sigma,\tau}\left(\sum_k (p_{m+1}^k - p_m^k)^2 | v_m\right) \geq \eta I\left((i_m, j_m) \in R, p_m \in \Delta_\delta(K)\right)$$

$\{p_m\}$ being a $\mathscr{F}_m$-martingale with values in $\Delta(K)$, $E_{\sigma,\tau}(\sum_{m=1}^n \sum_k (p_{m+1}^k - p_m^k)^2)$ is uniformly bounded by some constant C. Thus

$$\begin{aligned} C &\geq E_{\sigma,\tau}\left(\sum_{m=1}^{\infty} \sum_k I((i_m, j_m) \in R) I(p_m \in \Delta_\delta(K))(p_{m+1}^k - p_m^k)^2\right) \\ &= E_{\sigma,\tau}\left(\sum_{m=1}^{\infty} I((i_m, j_m) \in R) I(p_m \in \Delta_\delta(K)) E\Big(\sum_k (p_{m+1}^k - p_m^k)^2 | \mathscr{G}_m\Big)\right) \\ &\geq \eta E_{\sigma,\tau}\left(\sum_{m=1}^{\infty} I((i_m, j_m) \in R) I(p_m \in \Delta_\delta(K))\right). \end{aligned}$$

Note that $\sum_{m=1}^{\infty} I((i_m, j_m) \in R) I(p_m \in \Delta_\delta(K)) \geq LI(T_L < \theta)$ for any L so that

$$C \geq \eta L E_{\sigma,\tau}(I(T_L < \theta))$$

for any L. Hence, for any $\varepsilon > 0$, there exists M such that, for any pair of strategies in $\Gamma(p)$,

$$E_{\sigma,\tau}(I(T_M < \theta)) < \varepsilon/4. \tag{11}$$

g) End of proof

Given $\varepsilon > 0$, choose $\delta = \varepsilon/4$, and let M be determined by ε, η and inequality (11). Then let $\varepsilon_0 = \varepsilon/4M$ and finally define $N(\varepsilon) = 4n_0(M+1)/\varepsilon$. By (9) and (10), (σ^*, τ^*) is an ε-uniform equilibrium in $\Gamma(p)$ with payoff $(a(0), b(0))$. ■

5. Comments and open problems

The proof by Forges (1982) in the zero-sum case with random signals uses an auxiliary game to construct an operator H on (continuous) functions on $\Delta(K)$, such that if a player can guarantee some function u, he can also guarantee $H(u)$: the value of the game where the revealing entries are absorbing with payoff induced by u at the relevant posterior. Then an increasing sequence of functions $(u_n)_{n=1}^{\infty}, u_{n+1} = \max(H(u_n), u_n)$, and dually functions w_n are defined. One proves that the limit of both sequences are the same and define the value of the infinitely repeated game (see also Mertens and Zamir 1971–1972). Obviously this approach relies on the zero-sum aspect through the monotonicity of the value operator and therefore cannot be extended to the non-zero sum case. On the other hand, our construction gives an alternative proof to Forges's result. One should note that an argument related to the finite number of "interior jumps" of the martingale of posteriors was mentioned in the concluding remarks of her paper.

The result of the present paper extends easily to the case where H and K are countable. However finiteness assumptions on I and J are crucial for (3) to hold.

To conclude, recall that this research is part of a general program which aims to characterize the information structures for which equilibrium payoffs exist.

For two person games with lack of information on one side, existence has been recently proved by Simon, Spiez and Torunczyk (1995).

Note that in the framework of lack of information on both sides, already in the zero sum case the value may not exist, see Aumann and Maschler (1995).

This paper provides a positive answer for a class of two person non-zero sum games with symmetric incomplete information. A proof for the n person case would follow in the same way from the proof of existence of equilibria for n person games with absorbing states.

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