"Big Match" with Lack of Information on One Side (III)

Sylvain Sorin and Shmuel Zamir

Abstract: We prove the existence of a minmax for a class of stochastic games with incomplete information on one side by using an auxiliary one shot game.

1. Introduction: We study here a new specific class of two person zero-sum stochastic games with lack of information on one side; we assume independence and one non absorbing state. The first hypothesis means that the state space can be represented as $K \times L$, the initial probability being on K (k is fixed from then on) and the transition on L being independent of k. Hence the game can be viewed as a "stochastic game with vector payoffs". (The second assumption is now clear).

A first approach appeared in Sorin [8], [9] studying "Big Match"- type games (i.e. where one line or one column is absorbing, see Blackwell and Ferguson [2]) but preliminary analysis of some other cases may still be needed before getting an idea of the general kind of strategies involved.

However interesting properties and conjectures are already available:

First, in the two papers quoted above, it was observed that $max\,min = lim\,v_n$, i.e. that the informed player could do as well in the infinitely repeated game than in long finite games and it was conjectured that this relation holds true in all stochastic games with lack of information on one side. In fact it is probably true in a much more general framework, namely repeated games with the following property (P): player 1 is more informed than player 2. This follows from two recent results of Mertens [4], see also Melolidakis [3]:

i) Given any general repeated game there exists a natural compact state space where the original game can be represented as a stochastic game such that finite and discounted values coincide.

T. E. S. Raghavan et al. (eds.), Stochastic Games and Related Topics, 101–112.

ii) Moreover under (P) one can mimic strategies of player 1 (and best replies of player 2) in the stochastic game to get similar properties in the original game. Assuming the stochastic game to have a value (which seems to be a reasonable conjecture) one then obtains the existence of $max\,min$, $lim\, v_n$ and $lim\, v_\lambda$ in the original game and their equality.

These facts explain why we will be concerned with the other aspect of the problem, namely existence and characterization of the minmax. As in the previous articles the minmax will appear as the value of a (one shot) auxiliary game with lack of information on one side: somehow the infinitely repeated game can be "normalized" when looking at strategies of player 2 and best replies of player 1. Note that similar tools were introduced and used by Mertens and Zamir [6], see also Waternaux [10], [11], to study games with incomplete information and no signals.

Finally we will provide (when there are two states of nature) a simple geometric characterization of the minmax based on the orthants that player 2 can approach in the game with vector payoffs and absorbing states. The aim is obviously to get, in this framework, a dual (geometric/analytic) necessary and sufficient condition for approachability of convex sets, as follows from Blackwell [1] in the "deterministic" case.

2. The Game: Let $G^k, k \in K$ be a finite set of 2×2 payoff matrices of the form $G^k = \begin{pmatrix} a^{*k} & b^k \\ c^k & d^k \end{pmatrix}$, where the star $(*)$ denotes an absorbing payoff. We can assume $|G^k| \leq 1, \forall k \in K$. For each p, probability on K,$G(p)$ denotes the infinitely repeated game with the following common knowledge rules : k is chosen according to p, player 1 (the row player) is informed about k, players choose their moves simultaneously (full monitoring is assumed : at each stage, both players are told the previous moves) and the stage payoff is computed according to G^k.

Denote by I and J the set of moves of player 1 and player 2. As usual histories are finite sequences of couples in $I \times J$ and write H for the corresponding set. Strategies for player 1 (resp. 2) are transition probabilities from $K \times H$ to I (resp. from H to J) and are denoted by σ in Σ (resp. $\tau \in \mathcal{T}$). A couple (σ, τ) and k (or equivalently σ^k and τ) defines a probability on the product σ-algebra on the set of plays $H_\infty = (I \times J)^\infty$. To each play corresponds a stream of vector payoffs $(g_1, \ldots, g_n, \ldots)$,$\overline{g}_n$ denotes the average up to stage n and $\overline{\gamma}_n^p(\sigma, \tau)$ the corresponding expected payoff under p, σ and τ. We thus have: $\overline{\gamma}_n^p(\sigma, \tau) = \sum_k p^k \overline{\gamma}_n^k(\sigma^k, \tau)$ and $\overline{\gamma}_n^k(\sigma^k, \tau) = E_{\sigma^k, \tau}(\overline{g}_n^k)$.

Finally $\overline{v}(p)$ is the minmax of $G(p)$ if:

(i) $\forall \varepsilon > 0, \exists N, \exists \tau, \forall n \geq N, \forall \sigma, \quad \overline{\gamma}_n^p(\sigma, \tau) \leq \overline{v}(p) + \varepsilon$

(ii) $\forall \varepsilon > 0, \forall \tau, \exists N, \exists \sigma, \forall n \geq N, \quad \overline{\gamma}_n^p(\sigma, \tau) \geq \overline{v}(p) - \varepsilon.$

Remark: The value of the game $G(k)$ (corresponding to p concentrated on k) is easily seen to be:

$$\text{if } a^k \geq d^k \ : \ \text{med } (a^k, b^k, d^k)$$
$$\text{if } a^k \leq d^k \ : \ \text{med } (a^k, c^k, d^k)$$

and ε-optimal strategies can be chosen in the following set (say for player 1): always Top ($\widetilde{T}$), always Bottom ($\widetilde{B}$), always $(x, 1-x)$ i.i.d. with $x \in (0,1)$.

3. An Auxiliary Game: $\Gamma(p)$ is the (one shot) game in normal form defined by X^K (resp. Y) strategy set of player 1 (resp. 2) and payoff f where: $X = \overline{\mathcal{N}} \cup \{\partial\}$ ($\overline{\mathcal{N}}$ is the compactification $\mathcal{N} \cup \{\infty\}$ of the set of positive integers $\mathcal{N}$ and ∂ is some isolated point with $\partial > \infty$)

$$Y = \{0,1\}^{\mathcal{N}}$$

the payoff f is the average of the state payoff f^k,

$$f(x,y) = \sum_k p^k f^k(x^k, y)$$

with finally:

$$f^k(n,y) = a^k\Big(1 - \prod_0^{n-1} y'_m\Big) + \Big(\prod_0^{n-1} y'_m\Big)(y_n c^k + y'_n d^k) \text{ for } n \in \mathcal{N}$$
$$f^k(\infty,y) = a^k\Big(1 - \prod_0^{\infty} y'_m\Big) + \Big(\prod_0^{\infty} y'_m\Big) d^k$$
$$f^k(\partial,y) = a^k\Big(1 - \prod_0^{\infty} y'_m\Big) + \Big(\prod_0^{\infty} y'_m\Big) b^k$$

where $y = (y_0, \ldots, y_m, \ldots) \in Y$, y'_m denotes $1 - y_m$, and $\prod_0^{-1} = 1$.

Defining $\theta(y) = min\{(m; y_m = 1) \cup \{\infty\}\}$, it is clear that $\theta(y)$ determines the payoff, hence Y can also be written as $\overline{\mathcal{N}}$. Then one has, with $\xi \in X$:

$$f^k(\xi, m) = \mathrm{I}\{\xi \leq m-1\} d^k + \mathrm{I}\{\xi = m\} c^k + \mathrm{I}\{\xi > m\} a^k \text{ for } m \in \mathcal{N}$$
$$f^k(\xi, \infty) = \mathrm{I}\{\xi \leq \infty\} d^k + \mathrm{I}\{\xi = \partial\} b^k.$$

We write $\overline{\Gamma}(p)$ for the mixed extension of $\Gamma(p)$ where player 1's strategies are probabilities on (the borel subsets of) X^K (or as well K vector of probabilities on X, since f is decomposed on X^K), say χ in $\mathbf{X}^K$, and player 2's strategies are probabilities with finite support on Y, say Ψ in $\mathbf{Y}$.

Proposition 1: $\overline{\Gamma}(p)$ has a value $w(p)$.

Proof: For each $y, f^k(\cdot, y)$ is continuous on X. In fact either y corresponds to some m in $\mathcal{N}$ and f is constant on $x^k > m$, or to ∞ and f is constant on $\overline{\mathcal{N}}^K$. It follows that for (χ, Ψ) in $(\mathbf{X} \times \mathbf{Y})$:

$$F(\chi, \Psi) = \int f(x, y)\chi(dx)\Psi(dy)$$

is continuous in the first variable and affine in both. Moreover $\mathbf{X}^K$ is compact, hence by a corollary of Sion's theorem (see Mertens, Sorin and Zamir [5], Chapter 1), the game has a value (and player 1 has an optimal strategy).

4. The Result:

Theorem 2: $min\, max\, G(p)$ exists and $\overline{v}(p) = w(p)$.
The proof will follow from Lemmas 4 and 5.

Lemma 3: $\forall \Psi \in \mathbf{Y}$ there exists $z \in [0,1]^{\mathcal{N}}$ such that :

$$\int f(x, y)\Psi(dy) = f(x, z) \text{ for all } x \in X^K.$$

Proof: By the above remarks, Ψ can be described as the distribution of the stopping time θ. z_n is then just the conditional probability on the n^{th} factor, given $\{\theta \geq n\}$.

Lemma 4: w satisfies (i).

Proof: Given $\varepsilon > 0$, let Ψ be an $\varepsilon/4$ optimal strategy of player 2 for $w(p)$, and by Lemma 3, represent Ψ by some z in $[0,1]^{\mathcal{N}}$.

Define now τ, strategy of player 2 in $G(p)$ as: play z_0 i.i.d. until the first Top of player 1, then play z_1 i.i.d. until the second Top, then z_2, and so on.

Let also $\rho = \prod_{m=0}^{\infty} z'_m$ and define N such that:

$$\text{if } \rho = 0, \qquad \prod_{m=0}^{N-1} z'_m < \varepsilon/4$$

$$\text{and if } \rho > 0, \qquad \prod_{m=N}^{\infty} z'_m > 1 - \varepsilon/4.$$

Let us now majorize $\overline{\gamma}_n(\sigma, \tau)$ for $n \geq N$. It is enough to consider each $\overline{\gamma}_n^k(\sigma^k, \tau)$. Since τ is independent of the previous moves of player 2, we can assume that σ^k

has the same property (one can replace at each stage σ^k by its expectation w.r.t. τ on J without changing the payoff). Moreover we can consider best replies and assume σ^k pure.

It follows then that σ^k is completely described by the dates $M_1, \ldots, M_m, \ldots$ of the successive Top, and define $M_0 = -1$.

We thus obtain, for $n \in (M_m, M_{m+1})$:

$$\begin{aligned}\gamma_n^k(\sigma,\tau) &= E_{\sigma^k,\tau}(g_n^k) \\ &= a^k\Big(1 - \prod_{\ell=0}^{m-1} z'_\ell\Big) + \Big(\prod_{\ell=0}^{m-1} z'_\ell\Big)(z_m c^k + z'_m d^k) \\ &= f^k(m,z).\end{aligned}$$

Now for $n \geq M_N$, the expected stage payoff satisfies:
if $\rho = 0$, $|\gamma_n^k(\sigma,\tau) - a^k| \leq \varepsilon/2$, thus $|\gamma_n^k(\sigma,\tau) - f^k(\infty, z)| \leq \varepsilon/2$
if $\rho > 0$, either player 1 plays Bottom and one has:

$$|\gamma_n^k(\sigma,\tau) - (a^k(1-\rho) + \rho d^k)| \leq \varepsilon/2 \text{ by the choice of } N, \text{ thus:}$$
$$|\gamma_n^k(\sigma,\tau) - f^k(\infty, z)| \leq \varepsilon/2,$$

or player 1 plays Top and we obtain similarly:

$$|\gamma_n^k(\sigma,\tau) - f^k(\partial, z)| \leq \varepsilon/2.$$

It follows that every expected stage payoff, except at most $K \times N$ of them, corresponding to $M_m, m = 1, \ldots, N$, for each σ^k, is within $\varepsilon/2$ of a feasible payoff against Ψ in $\overline{\Gamma}(p)$.

Hence $n \geq 8KN/\varepsilon$ implies:

$$\overline{\gamma}^p(\sigma,\tau) \leq w(p) + \varepsilon.$$

Lemma 5: w satisfies (ii).

Proof: Consider first χ, optimal strategy of player 1 in $\overline{\Gamma}(p)$. Given $\varepsilon > 0, \chi$ and τ strategy of player 2 in $G(p)$, we shall define σ, strategy of player 1 in $G(p)$ by the following procedure: we will introduce a family, indexed by X, of "non revealing" strategies in $G(p)$, i.e. transition probabilities from H to I, say α_x. σ^k will then be: select some α_x according to the probability χ^k on X. Let η be the stopping time of reaching the absorbing entry:

$$\eta = min\{(m; i_m = \text{Top}, j_m = \text{Left}) \cup \{\infty\}\}$$

and define N such that

$$\chi^k(N) \leq \varepsilon/3 \text{ for all } k \in K.$$

For each $m \leq N$, define inductively strategies α_m and times L_m as follows: α_0 is: always Bottom.

Given α_0 and τ, let $t_n^0 = \mathrm{Prob}_{\alpha_0,\tau}(j_n = \text{Left})$ and L_0 be the first time ℓ where:

$$\sum_k p^k \chi^k(0)(t_\ell^0 c^k + t_\ell^{0'} d^k) \leq \inf_n \left\{ \sum_k p^k \chi^k(0)(t_n^0 c^k + t_n^{0'} d^k) \right\} + \varepsilon/3.$$

Define now α_1 as: Bottom up to stage L_0 (excluded), Top at that stage L_0, and always Bottom after.

Similarly given α_m, let $t_n^m = \mathrm{Prob}_{\alpha_m,\tau}(j_n = \text{Left} \mid \eta > L_{m-1})$ and let L_m be the first $\ell > L_{m-1}$ where:

(1) $\sum_k p^k \chi^k(m)(t_\ell^m c^k + t_\ell^{m'} d^k) \leq \inf_n \left\{ \sum_k p^k \chi^k(m)(t_n^m c^k + t_n^{m'} d^k) \right\} + \varepsilon/3.$

α_{m+1} is then α_m up to stage L_m (excluded), Top at that stage and Bottom thereafter.

Now for $m > N$, we first introduce a new stopping time L' and a non revealing strategy α' satisfying :

(2) $\pi = \mathrm{Prob}_{\alpha',\tau}(\eta \leq L') \geq \sup_{\alpha \in A} \mathrm{Prob}_{\alpha,\tau}(\eta < \infty) - \varepsilon/9.$

where A is the set of strategies that coincide with α_N up to L_{N-1} (included).

If $N < m \leq \infty$, let $\alpha_m = \alpha_\infty$: play α' up to stage L' (included) and then Bottom for ever.

Finally we define α_∂ as: play α' up to stage L' (included) and then always Top.

Let us now introduce z in $[0,1]^{\mathcal{N}}$ satisfying: $z_m = t_{L_m}$, for $m < N$, $z_N = u$ where $\pi = 1 - \left(\prod_{m=0}^{N-1} y_m'\right) u'$ (note that $1 \geq \pi \geq 1 - \prod_{m=0}^{N-1} y_m'$), $z_m = 0$, for $m > N$, and we shall prove that for $n \geq L'$:

(3) $\gamma_n^p(\sigma,\tau) \geq \int f(x,z)\chi(dx) - 2\varepsilon/3.$

In fact we can decompose the above payoff on the events $\{\alpha_x \text{ is played }\}$, with x in X, so that:

$$\gamma_n^p(\sigma,\tau) = \sum_k p^k \gamma_n^k(\sigma^k,\tau) = \sum_k \sum_x p^k \, \mathrm{Prob}_{\sigma^k}(\alpha_x)\gamma_n^k(\alpha_x,\tau) = \sum_x \varphi_n(x,\tau).$$

For $m \leq N$ we obtain:

$$\sum_k p^k \chi^k(m) \Big\{ \Big(1 - \prod_0^{m-1} y'_\ell \Big) a^k + \prod_0^{m-1} y'_\ell (t_n^m c^k + t_n^{m'} d^k) \Big\}$$
$$\geq \sum_k p^k \chi^k(m) \Big\{ \Big(1 - \prod_0^{m-1} y'_\ell \Big) a^k + \prod_0^{m-1} y'_\ell (y_m c^k + y'_m d^k) \Big\} - \varepsilon/3$$
$$\geq \sum_k p^k \chi^k(m) f^k(m, y) - \varepsilon/3.$$

For $N < m \leq \infty$, we get:

$$\sum_k p^k \chi^k(m) \Big\{ \text{Prob}_{\alpha_m, \tau}(\eta \leq n) a^k + (1 - \text{Prob}_{\alpha_m, \tau}(\eta \leq n))(t_n^m c^k + t_n^{m'} d^k) \Big\}$$
$$\geq \sum_k p^k \chi^k(m)(\pi a^k + (1 - \pi) d^k) - \varepsilon/3$$

since the choice of α' and L' implies $(1 - \text{Prob}_{\alpha_m, \tau}(\eta \leq n)) t_n^m \leq \varepsilon/9$.
Similarly, when α_∂ is used the payoff is at least:

$$\sum_k p^k \chi^k(\partial)(\pi a^k + (1 - \pi) b^k) - \varepsilon/3.$$

It follows that for all $m \in X$, $m \neq N$, $\varphi_n(m, \tau) \geq \sum_k p^k \chi^k(m) f^k(m, z) - \varepsilon/3$. Since, by the choice of N, $|\varphi_n(N, \tau)|$ as well as $|\sum_k p^k \chi^k(N) f^k(N, z)|$ are bounded by $\varepsilon/3$, we obtain (3) by summing.
Hence $n \geq 6L'/\varepsilon$ implies:

$$\overline{\gamma}_n^p(\sigma, \tau) \geq w(p) - \varepsilon.$$

This achieves the proof of Theorem 2.

5. A Geometric Approach: We shall here obtain the minmax by describing its supporting hyperplanes; equivalently, we will construct a convex set S in $\mathcal{R}^K$ with: $\overline{v}(p) = min_{s \in S} \langle p, s \rangle$.

5.1. General results: (see Sorin, [8], p.187)

We first need some definitions:

Player 1 can *force a set* S in $\mathcal{R}^K$, if for every $\varepsilon > 0$ and every τ there exists a strategy σ and a number N, such that $n \geq N$ implies that $\overline{\gamma}_n(\sigma, \tau) = \{\overline{\gamma}_n^k(\sigma, \tau)\}_{k \in K}$ belongs to S_ε^+, with $S_\varepsilon^+ = \{t \in \mathcal{R}^K;$ there exists $s \in S, t^k \geq s^k - \varepsilon, \forall k \in K\}$. (Write S^+ for S_0^+).

Player 2 can *guarantee a point* M in $\mathcal{R}^K$, if for every $\varepsilon > 0$ there exists a strategy τ and a number N , such that for every σ, $n \geq N$ implies: $\overline{\gamma}_n^k(\sigma, \tau) \leq M^k + \varepsilon$, for all k.

It is easy to see that the set S_2 of points that player 2 can guarantee is convex. On the other hand if $H(p, u)$ denotes the half space $\{t \in \mathcal{R}^K; \langle p, t \rangle \geq u\}$ (with p probability on K) and player 1 can force $H(p, g(p))$ for all p, then player 1 can force $H(p, \mathrm{Cav} g(p))$. Let then S_1 be the intersection of all half spaces $H(p, u)$ that player 1 can force. We now obtain:

Proposition 6: $\overline{v}(p)$ exists and equals $min_{s \in S}\langle p, s \rangle \iff S_1 = S_2 (= S)$ (S is then called the minmax set).

5.2. Description of S:

We assume from now on $\#K = 2$.

We denote by A (resp. B, C, D) the point (a^1, a^2) and define $V = (v^1, v^2)$ where v^k is the value (minmax would suffice) of $G(k)$. We also introduce $Q = (a^1, c^2), P = (v^1, d^2)$ and similarly $Q' = (a^2, c^1), P' = (d^1, v^2)$. Given two points M and N, let $M \vee N = (max\{m^1, n^1\}, max\{m^2, n^2\})$ and finally $[M, N]$ is the line segment MN.

Lemma 7: Player 1 can force $\{V\}$ (I), $[C, D]$ (II), $[A, B \vee D]$ (III), $[Q, P]$ (IV) and $[Q', P']$ (V).

Proof: (I) and (II) are clear. For (III), player 1 uses first a strategy that achieves the maximal probability of getting the absorbing payoff and plays then either Top or Bottom for ever (cf. (2) in Lemma 5). As for (IV), player 1 plays always Bottom in game 2 and Bottom in game 1 until the stage where the expected (non absorbing) payoff in game 2 is minimal. He plays then Top (in game 1) and thereafter optimally. The payoff in game 1 will then be some $ta^1 + (1-t)v^1$ in game 1 and in game 2 some average of expressions like $t_n c^2 + (1-t_n)d^2 \geq tc^2 + (1-t)d^2$, (cf. (1) in Lemma 5).

Lemma 8: Player 2 can guarantee $(\alpha) B \vee D, (\beta)(xC + x'D) \vee A$ for all $x \in [0, 1]$. Moreover, (γ) if player 2 can guarantee M he can also guarantee $(xC + x'D) \vee (xA + x'M)$.

Proof: (α) and (β) are clear. As for (γ), player 2 plays $(x, 1-x)$, i.i.d. until player 1 plays Top. He then approaches M.

These strategies are sufficient to obtain the following result:

Proposition 9: Let S be the intersection of the half spaces $H(p, u)$ containing one of the sets T, $T = I, \dots, V$. Then S is the minmax set.

Proof: It is enough to prove that the extreme points of the frontier of S belong to S_2.

1) If $D \in A^+$, player 2 can approach $B \vee D(\alpha)$ and $A(\beta)$; hence $S = [A, B \vee D]^+$.
2) If $A \in D^+$, either $C \in D^+$ or $D \in C^+$ and $S = V^+$; or suppose $c^1 > d^1$ and let x satisfy $xc^2 + (1-x)d^2 = v^2$. Then $E = xC + x'D$ belongs to S_2 and $S = [D, E]^+$.
3) In the remaining case we can assume $a^1 > d^1$ and $a^2 < d^2$.

The analysis is done by considering the different values of B and C and can be reduced to the 2 following configurations :

First case:

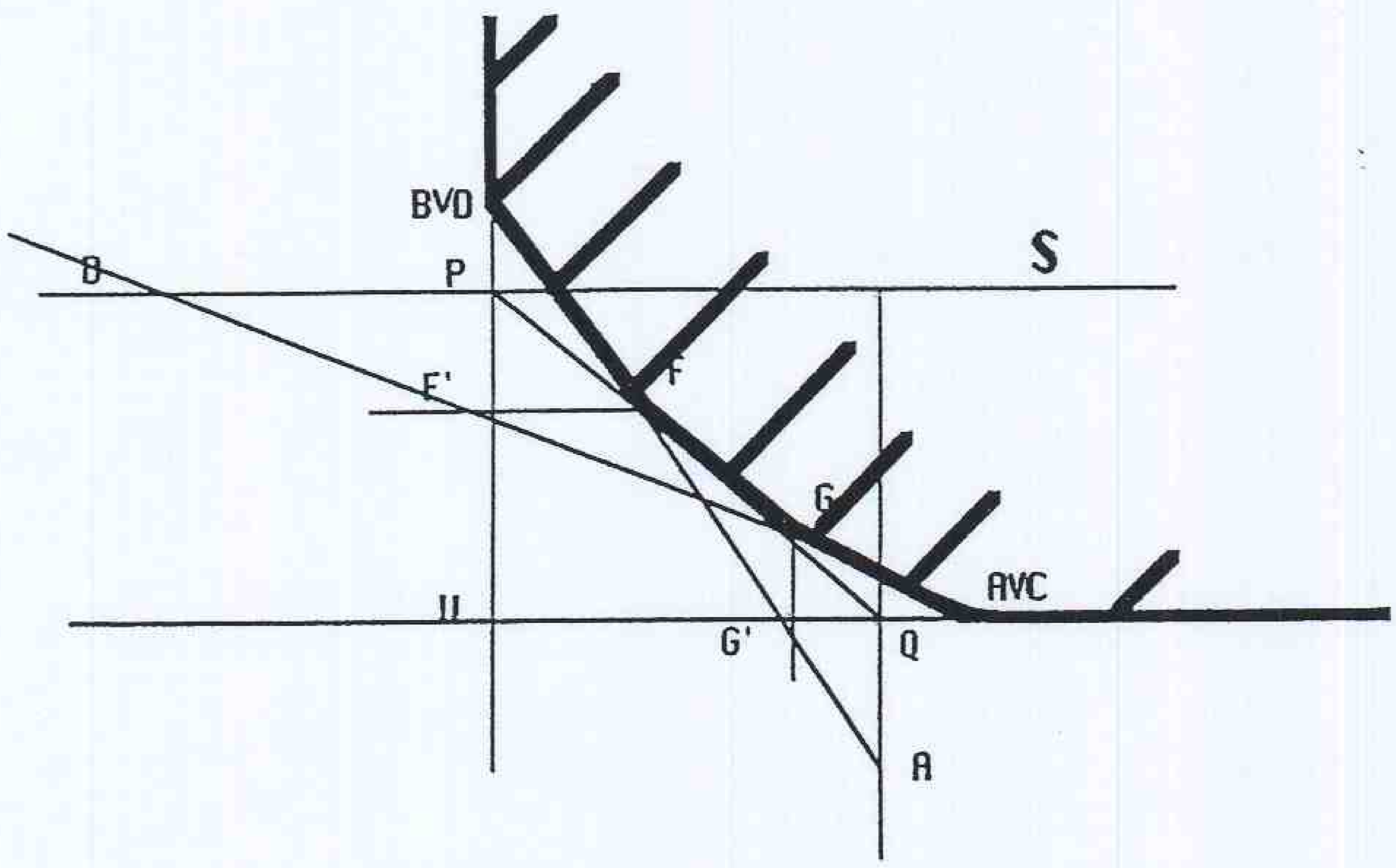

Figure 1

Player 2 can approach $B \vee D(\alpha)$ and $C \vee A(\beta)$.

For F, write $F = xA + x'B \vee D = xQ + x'P$, hence $F' = xC + x'D$ so that using $M = B \vee D$ in (γ), $F \in S_2$.

Similarly for G, write $G = yC + y'D = yQ + y'P$, so that $G' = yA + y'B \vee D$ and as above $G \in S_2$.

Second case:

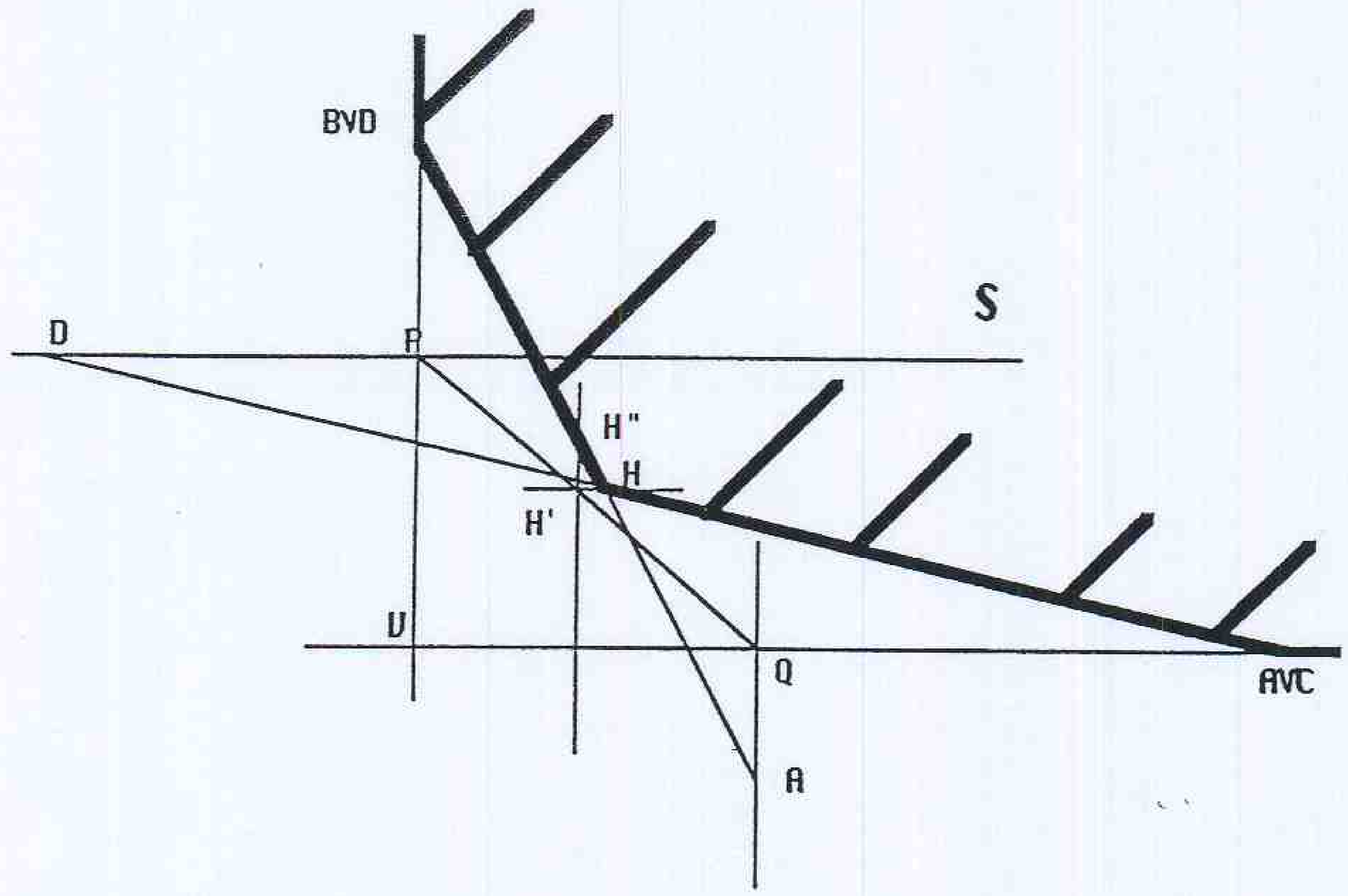

Figure 2

Here $H = xC + x'D, H' = xP + x'Q, H'' = xA + x'B \vee D$, so that H can be written as $xA + x'H_1$ with $H_1 \in [H, B \vee D]$. Write $x_0 = x$ and note that the same decomposition can now be done on H_1 leading to some x_1 and H_2 and so on. For some n, H_n will be be within ε of $B \vee D$ hence : to play $(x_j, 1 - x_j)$ i.i.d. after j moves Top of player 1 ($j < n$), and to guarantee $B \vee D$ after n Top is the required strategy.

Remarks: The above analysis indicates that for $\#K = 2$, player 1 can be restricted to use strategies with support on $\{\alpha_x\}$, $x = 0, 1, \infty, \partial$. Considering then only best replies of player 2, it follows that $\overline{v}(p)$ is the value of a (one-shot) matrix game with lack of information on one side (hence, as in Proposition 9, piece-wise linear).

On the other hand it is easy to see that one cannot restrict player 2 to use strategies in $\overline{\Gamma}(p)$ with θ uniformly bounded or ∞. In other words, given $\varepsilon > 0$, the number $N(\varepsilon)$ of "exceptional moves" (like in the proof of Proposition 9, second case) needed before using some i.i.d. strategy cannot be uniformly bounded.

A very similar, but simpler structure, is obtained when only one entry is non absorbing. When the absorbing states are on the diagonal, the minmax is again the value of some auxiliary game in normal form with lack of information on one side (Sorin, [9]).

Acknowledgements: It is a pleasure to thank J.F. Mertens for nice comments.

References

[1] *Blackwell, D.* (1956). An Analog of the Minimax Theorem for Vector Payoffs *Pacific Journal of Mathematics*, **6**, 1-8.

[2] *Blackwell, D. and T. S. Ferguson* (1968). The Big Match, *Annals of Mathematical Statistics*, **39**, 159-163.

[3] *Melolidakis, C.* (1984). Ph.D. Thesis, UCLA.

[4] *Mertens, J.F.* (1987). Repeated Games, *Proceeding of the International Congress of Mathematicians, Berkeley* (1986), 1528-1577.

[5] *Mertens, J.F., S. Sorin and S. Zamir* (1987). "Repeated Games", book to appear.

[6] *Mertens, J.F. and S. Zamir* (1976). On a Repeated Game without a Recursive Structure, *International Journal of Game Theory*, **5**, 173-182.

[7] *Sorin, S.* (1984). "Big match" with Lack of Information on One Side (Part I), *International Journal of Game Theory*, **13**, 201-255.

[8] *Sorin, S.* (1985). "Big match" with Lack of Information on One Side (Part II), *International Journal of Game Theory*, **14**, 173-204.

[9] *Sorin, S.* (1987). More on the 'Big match' with Lack of Information on One Side, manuscript.

[10] *Waternaux, C.* (1983). Solution for a Class of Games without Recursive Structure, *International Journal of Game Theory*, **12**, 129-160.

[11] *Waternaux, C.* (1983). Minmax and Maxmin of Repeated Games without a Recursive Structure, Core D.P. 8313.

Sylvain Sorin
Mathematiques
Université Louis Pasteur
7 Rue René Descartes
67084 Strassbourg
France

Shmuel Zamir
Department of Economics
University of Pittsburgh
Pittsburgh, PA 15260
U.S.A.

Stochastic G
and Positive

Abstract: W
(SGLIOS) und
probability. Th
for the informe
control under i
probabilities on

1. Introductio
stop probabiliti
optimal station
were generalize
In this paper
informational c
true state of nat
but he is inform
with Lack of In
One way to c
use of an updat
given the history
may restrict his
conditional prob
This general n
other cases, it
"learning" (e.g.

T. E. S. Raghavan et al.