SYMMETRIC INCOMPLETE INFORMATION GAMES AS STOCHASTIC GAMES

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Abstract. The purpose of this chapter is to show how games with incomplete information in fact reduce to stochastic games, as long as the information is symmetric among the players. The new state space corresponds to the beliefs on the space of unknown parameters.

1. Introduction

Stochastic games and incomplete information games have long been considered two quite distinct fields. In the first case the state is known to the players but evolves along the play, while in the second it remains the same but is partially unknown to some of the players. On the other hand, at least in the zero-sum case, it is quite clear (via the recursive formula) that the beliefs of the uninformed player hold the role of a state variable adapted to the play of the game. When the information is symmetric among the players, an exact reduction to a stochastic game on beliefs is available. Further relations will be exhibited in [12], [13] and a general approach is provided in [1].

The simple deterministic case is considered first, then the general random case is presented and the analysis is finally extended to stochastic games with symmetric incomplete information structure.

2. Deterministic Case

The following model was introduced by Kohlberg and Zamir [5].

K is a finite set of states of nature called parameters. For each k in K, G^k is a finite strategic form game played by I players with action spaces

A^i $(i \in I)$. In addition, "signalling functions" ℓ^k defined on $A = \prod_i A^i$ with value in some signal space Ω are given.

To describe the associated repeated game we have to specify the initial information of the players on the parameter and the additional information gathered along the play on the parameter and on the previous moves. The crucial feature of the present model is that both aspects are symmetric among the players.

Given a probability p on K, the game $\Gamma(p)$ is played as follows. The parameter k is chosen once and for all according to p but is not transmitted to the players. The game is played an infinite number of stages. At stage n, player i chooses $a_n^i \in A^i$ and a_n is the profile $\{a_n^i\}$ in A. The vector payoff in $\mathbb{R}^I$ at stage n is thus $r_n = G^k(a_n)$ but is not announced. Rather the players are told the "public signal" $\omega_n = \ell^k(a_n)$. It is assumed that the signal contains all the information of the players at that stage and that perfect recall holds; hence symmetric information implies that the signal contains the moves, i.e., $a \neq a'$ implies $\ell^k(a) \neq \ell^{k'}(a')$.

For every profile a, the set of feasible signals is $\ell(a) = \{\ell^k(a), k \in K\}$ and the signal observed induces a partition of K. For any signal ω, let $K(\omega)$ denote the set of k's compatible with it and let $p(\omega)$ be the corresponding conditional probability on K. A profile of actions a is *non-revealing* (at the probability p with support K) if $\omega = \ell^k(a)$ is independent of k: then $K(\omega) = K$ and $p(\omega) = p$. Note that if $K(\omega) \neq K$, the cardinality of the parameter space is strictly decreasing and this will allow for an induction procedure on it.

The following example deals with a zero-sum game and is taken from [6]. The state space is $K = \{L, M, R\}$ and the initial probability p on K is uniform. The payoffs are given by

0	0
0	4

L

0	−2
2	0

M

0	0
−4	−2

R

and the signals by

lm	l
P	Q

L

lm	mr
P	Q

M

r	mr
P	Q

R

The value of each matrix game is obviously 0. If $a = (Top, Left)$ is played, the signal ω will be lm with probability 2/3 and r with probability 1/3.

In this second case ($\omega = r$), the game R is revealed and one can assume the payoff from then on to be 0. In the first case ($\omega = lm$), the game from this stage on belongs to the set of states $\{L, M\}$ with initial prior $(1/2, 1/2)$. The moves $(Top, Left)$, $(Bottom, Left)$ and $(Bottom, Right)$ are non-revealing; hence they induce the expected payoffs 0, 1 and 2 respectively. The move $(Top, Right)$ is completely revealing and thus leads to the payoff $0 = (1/2)\mathtt{val}(L) + (1/2)\mathtt{val}(M)$. Hence, the reduced game (following the move $a = (Top, Left)$ and signal $\omega = lm$) can be analyzed through the following absorbing game

0	0^*
1	2

(where as usual a star * denotes an absorbing payoff). The value of this absorbing game is 1.

A similar analysis applies if $(Top, Right)$ is played.

Finally, if player 1 plays *Bottom*, there is no change in information on K and the payoff is the expectation. The initial game is thus asymptotically equivalent to the following:

$((2/3)\times 1 + (1/3)\times 0)^*$	$((2/3)\times(-1) + (1/3)\times 0)^*$
$-2/3$	$2/3$

which is again a stochastic game with absorbing states; hence it has a value [4]; see also [14].

A similar reduction was proved to apply to any game in the zero-sum deterministic case by Kohlberg and Zamir [5] and then extended by Neyman and Sorin [9] to the non-zero-sum setup.

Property 1 *The analysis of a game with symmetric and deterministic information reduces to the analysis of a finite sequence of absorbing games.*

Proof. The proof is by induction on the number of states $|K|$. Define a game $\Gamma'(p)$ as a repeated game with absorbing states and standard signalling as follows:
- if a is non-revealing at p, the state p does not change, the payoff is the average $\sum_k p^k G^k(a)$ and a is announced;
- otherwise, with probability $\sum_{\{k;\ell^k(a)=\omega\}} p^k$, the new state is $p(\omega)$ and is absorbing. The induction hypothesis implies that the game $\Gamma(p(\omega))$ reduces to an absorbing game.

The state space is thus the (finite) set of posterior probabilities that can be generated through the signals starting from p. ∎

In the zero-sum case this leads to the following:

Proposition 1 (Kohlberg and Zamir [5]) *Any zero-sum repeated game with symmetric and deterministic information has a value.*

Proof. It follows from Property 1 and the existence of a value for zero-sum absorbing games [4].

Explicitly, the payoff of the absorbing game $\Gamma'(p)$ is given by

$$G'(p)(a) = \begin{cases} \sum_k p^k G^k(a) & \text{if } a \text{ is non-revealing,} \\ \left(\sum_{\omega\in\ell(a)} \sum_{k\in K(\omega)} p^k v(p(\omega))\right)^* & \text{otherwise,} \end{cases}$$

where $v(p(\omega))$ is defined by induction to be the value of the game starting at $p(\omega)$. (Recall that if $p(\omega)$ differs from p, it belongs to the boundary of $\Delta(K)$.) ∎

In the non-zero-sum case the corresponding result is

Proposition 2 (Neyman and Sorin [9]) *The existence of equilibrium payoff in I-person absorbing games implies the existence of equilibrium payoff in I-person repeated games with symmetric and deterministic information. In particular, existence of equilibrium payoff holds for two- and three-person games.*

Proof. The proof is again by induction on the size of the support of p. If $p(\omega)\neq p$, the support of $p(\omega)$ is a proper subset of the support of p. Therefore, $\Gamma(p(\omega))$ has an equilibrium payoff, say $g(\omega)$. Now let the payoff of the I-person absorbing game $\Gamma'(p)$ be defined by

$$G'(p)(a) = \begin{cases} \sum_k p^k G^k(a) & \text{if } a \text{ is non-revealing,} \\ \left(\sum_{\omega\in\ell(a)} \sum_{k\in K(\omega)} p^k g(\omega)\right)^* & \text{otherwise.} \end{cases}$$

Assume that $\Gamma'(p)$ has an equilibrium payoff. We claim that it is also an equilibrium payoff of $\Gamma(p)$: in fact it is clear that playing the equilibrium strategies in $\Gamma'(p)$ as long as an absorbing state is not reached and then, if the revealing signal ω ($p(\omega) \neq p$) is observed, playing the equilibrium strategies inducing the equilibrium payoff $g(\omega)$ in $\Gamma(p(\omega))$, will define equilibrium strategies.

Explicitly, given such a profile of strategies as above, σ, and an alternative strategy τ^i of player i, one has

$$E_{\sigma^{-i},\tau^i}\left(\sum\nolimits_{m=1}^{n} r_m^i\right) = E_{\sigma^{-i},\tau^i}\left(\sum\nolimits_{m=1}^{\theta} r_m^i + \sum\nolimits_{m=\theta+1}^{n} r_m^i\right),$$

where θ is the stopping time corresponding to the entrance in an absorbing state in $\Gamma'(p)$. Since σ induces an equilibrium after θ in the reduced game, there exists N such that if the number of remaining stages is large enough, $(n-\theta) \geq N$, one has:

$$E_{\sigma^{-i},\tau^i}(\sum_{m=\theta+1}^{n} r_m^i|\mathcal{H}_\theta) \leq (n-\theta)(g_\theta^i + \varepsilon)$$

and

$$E_\sigma(\sum_{m=\theta+1}^{n} r_m^i|\mathcal{H}_\theta) \geq (n-\theta)(g_\theta^i - \varepsilon),$$

where g_θ^i is the absorbing payoff of player i in $\Gamma'(p)$ from stage θ on. So that if C is a bound on the payoffs one obtains

$$E_{\sigma^{-i},\tau^i}(\sum_{m=1}^{n} r_m^i) \leq E_{\sigma^{-i},\tau^i}(\sum_{m=1}^{\theta} r_m^i + (n-\theta)g_\theta^i) + n\varepsilon + NC.$$

Now use the fact that σ is up to θ an equilibrium in $\Gamma'(p)$. Hence, for n large enough,

$$E_{\sigma^{-i},\tau^i}(\sum_{m=1}^{\theta} r_m^i + (n-\theta)g_\theta^i) \leq E_\sigma(\sum_{m=1}^{\theta} r_m^i + (n-\theta)g_\theta^i) + n\varepsilon.$$

Finally,

$$\begin{aligned} E_{\sigma^{-i},\tau^i}(\sum_{m=1}^{n} r_m^i) &\leq E_\sigma(\sum_{m=1}^{\theta} r_m^i + (n-\theta)g_\theta^i) + 2n\varepsilon + NC \\ &\leq E_\sigma(\sum_{m=1}^{n} r_m^i) + 3n\varepsilon + 2NC, \end{aligned}$$

which is the equilibrium condition.

The second assertion of Proposition 2 then follows from the existence of a uniform equilibrium payoff for two-person absorbing games [15] (see also [7], pp. 406-408), and recently for the three-person case [11]. ∎

3. Random Case

We now consider the random case where for each (k, a), $\ell^k(a)$ is a distribution on a finite set of signals Ω. The symmetric information hypothesis requires that if a differs from a', any $\ell^k(a)$ and $\ell^{k'}(a')$ have disjoint supports.

The play of the game is similar to the deterministic case, but one cannot, in case of a revealing profile, start an induction on the size of the support of the beliefs. However, a similar notion of revelation will be useful.

Let $\tilde{q}(p, a)$ be the distribution of the posterior probability on K, when the prior is p and the vector of moves played by the players is a. Explicitly, let $\ell^p = \sum_k p^k \ell^k$ and define a function $q : \Delta(K)\times A\times\Omega \to \Delta(K)$ satisfying

$$\ell^p(a)(\omega)q^k(p, a, \omega) = p^k \ell^k(a)(\omega).$$

For each $(p,a) \in \Delta(K) \times A$, $\tilde{q}(p,a)$ has the following distribution:

$$Prob(\tilde{q}(p,a) = q(p,a,\omega)) = \ell^p(a)(\omega) = Prob(\omega|p,a).$$

In words, $q(p,a,\omega)$ is the posterior distribution on K given the signal ω and $\ell^p(a)(\omega)$ its probability.

The subset of A for which $\tilde{q}(p,a)$ is the constant p consists of the *non-revealing* entries at p: the signal ω is uninformative and the posterior does not change.

We will now provide two approaches that use quite different tools. The first one solves the zero-sum case while the second one also applies to the non-zero-sum case.

3.1. ZERO-SUM CASE

This subsection follows [2]. As above, the proof relies on an auxiliary game. Given any real function f defined on $\Delta(K)$, introduce the following absorbing game with payoff

$$D(f,p)(a) = \begin{cases} \sum_k p^k G^k(a) & \text{if } a \text{ is non-revealing at } p, \\ \Big(E(f(\tilde{q}(p,a))\Big)^* & \text{otherwise,} \end{cases}$$

and denote by $T(f,p)$ its value. The main result is then

Theorem 1 (Forges [2]) *The mapping $T : f \mapsto T(f,.)$ has a unique continuous fixed point v. $v(p)$ is the value of $\Gamma(p)$.*

The proof is by induction on the number of states $|K|$. The basic steps of the proof are as follows. By the induction hypothesis the result is true on the boundary ∂ of $\Delta(K)$. I.e., there exists a continuous function w defined on the boundary ∂ such that $T(w,.) = w(.)$.

Lemma 1 *Let u be continuous on $\Delta(K)$ and be equal to w on the boundary. Then $T(u)$ is continuous on $\Delta(K)$ and equals w on the boundary.*

The delicate point is the continuity at the boundary. It relies on the fact that the value of an absorbing game does not change whenever a non-absorbing entry is replaced by an absorbing one with payoff equal to the value.

The next result is similar to the argument sketched in the previous section.

Lemma 2 *If player one can guarantee $u(p)$ in $\Gamma(p)$, $\forall p \in \Delta(K)$, he can also guarantee $T(u,p)$.*

One now uses the zero-sum aspect to generate a monotonic sequence of functions (u_n) defined on $\Delta(K)$. Player one can guarantee

$$u_0(p) = max_{r\in\partial}\{w(r) - \|G\|\|r-p\|_1), \text{ with } \|G\| = max_{k,a}|G^k(a)|.$$

Thus he can also guarantee

$$u_{n+1} = \max\{u_n, T(u_n)\}.$$

This defines an increasing sequence of continuous functions on $\Delta(K)$, equal to w on the boundary ∂ and converging to some $\underline{u}$. $\underline{u}$ is lowersemicontinuous, $\underline{u} = w$ on ∂, $\underline{u} \geq T(\underline{u})$ (since T is nonexpansive, hence continuous) and player one can guarantee $\underline{u}$.

Dual results obviously hold for player two with a function $\overline{u}$; hence in particular $\underline{u} \leq \overline{u}$. It thus remains to show

Lemma 3

$$\underline{u} \geq \overline{u}.$$

Proof. By contradiction let ρ in $\Delta(K)$ be an extreme point of the convex hull of the compact set where $\overline{u} - \underline{u}$ (which is u.s.c.) is maximal and equal to $\delta > 0$. Since ρ is the expectation of $\tilde{q}(\rho, a)$ for any a revealing at ρ, one obtains by the extremality condition

$$E((\overline{u} - \underline{u})(\tilde{q}(\rho, a))) < \delta.$$

However, for a non-revealing at ρ the payoffs are the same in $D(\overline{u}, \rho)$ and $D(\underline{u}, \rho)$; hence, for all a,

$$D(\overline{u}, \rho)(a) - D(\underline{u}, \rho)(a) \ < \delta,$$

so that $T(\overline{u}, \rho) - T(\underline{u}, \rho) < \delta$ and a fortiori $(\overline{u} - \underline{u})(\rho) < \delta$, a contradiction. ∎

Note that this proof is reminiscent of a similar construction for the proof of existence of $\lim v_n$ in two-person zero-sum games with incomplete information on both sides [8].

3.2. NON-ZERO-SUM CASE

In the current non-zero-sum framework the result will also be obtained by induction; however, one cannot rely on the monotonicity of the iterative functions since the correspondence from payoffs to equilibria is not monotonic.

In the previous section, ε-equilibrium strategies at stage m were functions only of the posterior at that stage. Here their computation will take into account the current value of the martingale of posterior probabilities and the number of stages where this value has changed. In fact, the finiteness assumption on A implies that, for any positive ε and any strategy pair, there is a finite number of jumps, say M, after which, with probability greater than ε, the martingale will be within ε of the boundary, hence the possibility of an induction analysis on the cardinality on K.

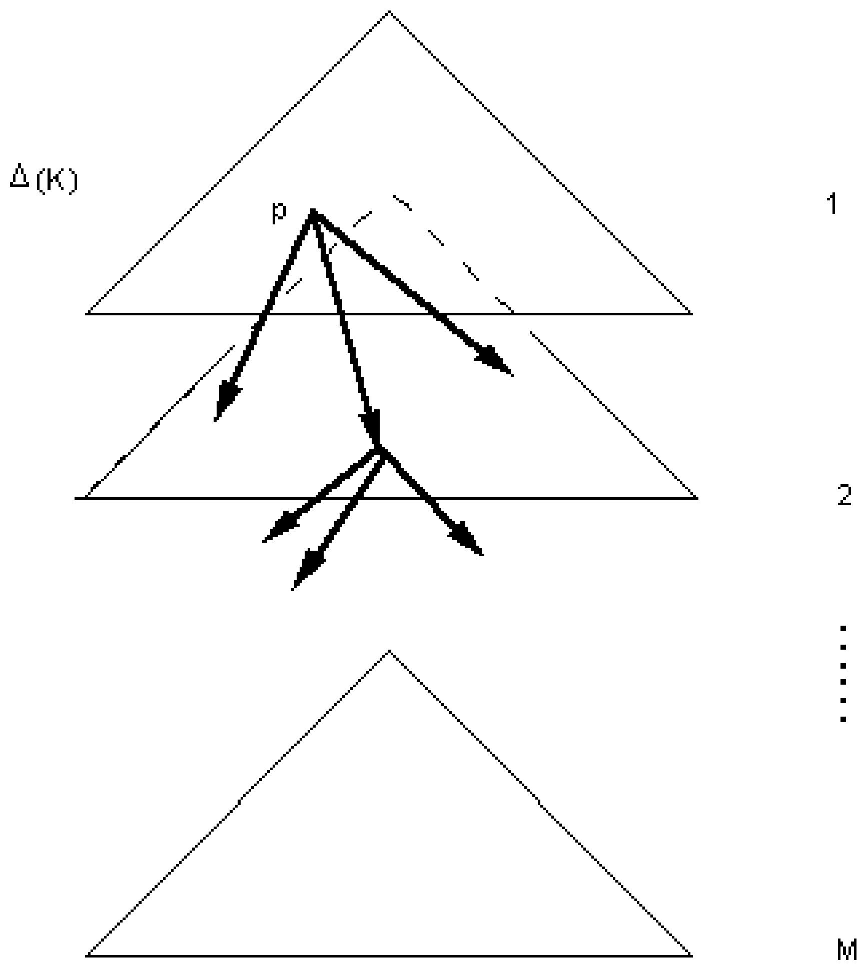

Figure 1. The stochastic process on the state space

Explicitly, the strategies will be constructed as follows. At the M-th jump, choose in the boundary of $\Delta(K)$ a closest point p_* to the current value p of the martingale and play according to an equilibrium payoff in $\Gamma(p_*)$ from this stage on. This defines a vector payoff $e(M,p)$. Inductively, vector payoffs $e(m,p)$ are defined on $\Delta(K)$ after m jumps ($m \leq M$). After

$m-1$ jumps, the players play, at p, equilibrium strategies in the stochastic game where the payoff is the expected stage payoff if the posterior does not change and is, after a jump, absorbing and equal to $e(m,p')$ where p' is the current posterior.

Hence, the state space will be a product $\Delta(K)\times\{1,2,\ldots,M\}$, as in Figure 1. If there is no splitting neither p nor the counter m changes. Otherwise, p evolves in the simplex and the counter increases by one.

Theorem 2 (Neyman and Sorin [10]) *Existence of equilibrium payoff in I-person absorbing games implies existence of equilibrium payoff in I-person games with symmetric incomplete information.*

Proof. Formally, the construction works as follows. For a fixed $\varepsilon > 0$, one looks for an element in E_ε for the game $\Gamma(p)$. Using the Lipschitz aspect of the payoffs w.r.t. p and the induction hypothesis, it is enough to deal with p at a distance greater than $\varepsilon/2$ from the boundary of $\Delta(K)$, say $p\in\Delta'$.

From the finiteness of A we deduce the existence of $\eta > 0$ such that for all p in Δ' and for all revealing a at p,

$$E(\sum_k(\tilde{q}^k(p,a) - p^k)^2) > \eta, \tag{1}$$

meaning that as long as p is not near the boundary, a revealing profile a induces a variance of the posteriors uniformly bounded below by a positive number. Call such a pair (p,a) a jump. Since the sum of the per-stage variation of the martingale of posteriors $\{p_n\}$, evaluated in L^2 norm, is bounded, namely for each k

$$\sum\nolimits_{n=1}^{\infty}(p^k_{n+1} - p^k_n)^2 \leq 1,$$

there exists an integer M such that the probability of the set of paths where more than M jumps occur before reaching the $(\varepsilon/2)$ boundary is less than $\varepsilon/2$.

Now introduce a new state space as $\bar{K} = \Delta(K) \times \{0,1,\cdots,M\}$ and define inductively a mapping α on $\bar{K}$ as follows. $\sigma(M,p)$ is an $(\varepsilon/2)$-uniform equilibrium strategy profile with vector payoff $\alpha(M,\,p)$ in the game $\Gamma(p)$ for $p \in \Delta\setminus\Delta'$ (which exists by the induction hypothesis on the number of elements in the support of p and the above remark). $\sigma(M,p)$ is arbitrarily defined for $p\in\Delta'$ and $\alpha(M,p)$ is the vector 0 there.

For $\ell = 0,1,\cdots,M-1$ and p near the boundary, namely $p\in\Delta\setminus\Delta'$, let $\alpha(\ell,p) = \alpha(M,\,p)$. Now, for $\ell = 0,1,\cdots,M-1$ and $p\in\Delta'$, define by backwards procedure a game with absorbing payoffs $G'(\ell,p)$ played on A

by

$$G'(\ell,p)(a) = \begin{cases} \sum_k p^k G^k(a) & \text{if } a \text{ is non-revealing at } p, \\ \{E(\alpha(\ell+1,\tilde{q}(p,a))\}^* & \text{otherwise.} \end{cases}$$

$G'(\ell,p)$ is an absorbing game with standard signalling and by hypothesis these games have ε-uniform equilibria strategies $\sigma(\ell,p)$ with payoffs $\alpha(\ell,p)$; hence the induction is well defined.

The claim is that $\alpha(0,p)$ belongs to $E_\varepsilon(p)$. First, introduce on the space of plays the stopping time W_ℓ, corresponding to the ℓ-th time that a revealing entry is played (the ℓ-th jump), $\ell = 1,\cdots,M$, and θ, the stopping time that corresponds to the entrance time in $\Delta(K)\setminus\Delta'$. Let $T_\ell = \min\,(W_\ell,\theta)$. The construction of a profile of strategies σ^* in $\Gamma(p)$ is as follows. σ^* coincides with $\sigma(0,p)$ until time T_1. Inductively, given the past history, σ^* follows $\sigma(\ell,p(\ell))$, from time $T_\ell+1$ until time $T_{\ell+1}$, $\ell = 1,\cdots,M$, where $p(\ell)$ is the posterior distribution on K given the past history h_{T_ℓ}. More precisely, for every subsequent history h, $\sigma^*(h_{T_\ell},h) = \sigma(\ell,p(\ell))\ (h)$.

Consider now a profile of strategies and the corresponding random path of the martingale of posterior distribution. If the number of jumps is less than M or if the boundary is reached, the previous computations apply and imply the equilibrium condition. Since the probability of the complementary event is less than $\varepsilon/2$, this ends the proof. ∎

4. Stochastic Games with Symmetric Incomplete Information

In fact the previous construction applies to a more general setting and this extension is due to [3].

Assume that, rather than dealing with repeated games G^k, each of them is actually a stochastic game played on some state space Ξ. k will refer to the uncertainty parameter while ξ in Ξ will be the stochastic state parameter.

The game evolves as follows. An initial public lottery p on K selects k and then G^k is played starting from ξ_1, which is publicly known. After each stage $m \geq 1$ a public random signal ω_m is announced, which reveals the profile of moves a_m and the new state parameter ξ_{m+1}. The distribution of ω_m depends upon k,ξ_m and a_m. As previously, the signals induce a (public) martingale $\tilde{p}$ of posterior distribution on K and one defines non-revealing profiles at (p,ξ) as those for which $\tilde{p} = p$. If $\tilde{\xi}$ denotes the new random state, the couple of parameters $\tilde{p},\tilde{\xi}$ is a random variable on $\Delta(K)\times\Xi$.

The family of auxiliary games is now defined on $\{1,...M\}\times\Delta(K)\times\Xi$ by the payoff

$$G'(\ell, p, \xi)(a) = \begin{cases} \sum_k p^k G^k(a) & \text{if } a \text{ is non-revealing at } (p, \xi) \\ & \text{and the new state is } \widetilde{\xi}, \\ \{E(\alpha(\ell+1, \widetilde{q}, \widetilde{\xi})\}^* & \text{if } a \text{ is revealing at } (p, \xi), \end{cases}$$

where, as in Section 3.2, α is constructed inductively as an equilibrium payoff. This defines a new stochastic game where absorbing states have been added. Note that if the initial games G^k are absorbing, the auxiliary game is also.

Thus one obtains

Theorem 3 (Geitner [3]) *If any I-person absorbing (resp. stochastic) game has an equilibrium payoff, any I-person absorbing (resp. stochastic) game with symmetric information has an equilibrium payoff.*

5. Comments

The main conclusion is that as long as the information is symmetric its evolution is similar to the state process in a stochastic game. The typical incomplete information features, how to use or reveal private information, occur only with differential information (see, e.g., [7]).

Two more technical remarks follow. The new state space is uncountable even in the case where the parameter space is finite; however, the state process is much simpler than in a general stochastic game since it is a martingale, and this enables us to reduce the analysis to the analysis of absorbing games.

The analysis extends, under regularity hypotheses, to countable or measurable signal spaces. However, the fact that A is finite is crucial to the proof in getting the "minimal amount of splitting" η in case of a revealing profile.

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