Errata for
“Higher-Dimensional Algebraic Geometry”

by Olivier Debarre

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page 2

• line 31, for

“A variety is an integral scheme”

read

“A variety is a geometrically integral scheme”

page 4

• line 20, for

“connected reduced”

read

“geometrically connected and reduced”

page 9

• line 12, for

“let C be a curve.”

read

“let C be an irreducible curve.”

page 11

• line 5, for

“the set of classes of effective 1-cycles.”

read

“generated by classes of effective 1-cycles.

If Y is another proper scheme, any morphism $\pi : X \to Y$ induces linear maps

$$\pi^* : N^1(Y)_{\mathbf{Z}} \to N^1(X)_{\mathbf{Z}} \qquad \text{and} \qquad \pi_* : N^1(X)_{\mathbf{Z}} \to N^1(Y)_{\mathbf{Z}}$$

defined by (see § 1.9)

$$\pi^*([D]) = [\pi^*(D)] \qquad \text{and} \qquad \pi_*([C]) = [\pi_*(C)] = \deg\bigl(C \xrightarrow{\pi} \pi(C)\bigr)\ [\pi(C)]$$

They satisfy the projection formula (see (1.5))

$$\pi^*(d) \cdot c = d \cdot \pi_*(c)$$

• line -13, for

“$\mathbf{Spec}(\pi_*\mathcal{O}_Y)$”

read

“$\mathbf{Spec}(\pi_*\mathcal{O}_X)$”

page 13

- line 6, for

“$X^0 = f^{-1}(Y^0)$”

read

“$X^0 = \pi^{-1}(Y^0)$”

page 22

- line -10, for

“the left-hand side of this equality is nonnegative”

read

“the left-hand side of this equality is nonpositive”

page 27

- line -11, for

“nine points”

read

“nine general points”

page 31

- line 10, as of

“Recall also...”

replace the end of the proof of Proposition 1.43 by

“ Each component of E has codimension 1 (1.40) hence, by shrinking Y, we may assume that Y and E are smooth and irreducible. Set $U_0 = X - \mathrm{Sing}(\overline{\pi(E)})$, so that the closure in U_0 of the image of $E \cap \pi^{-1}(U_0)$ is smooth, of codimension at least 2. Let $\varepsilon_1 : X_1 \to U_0$ be its blow-up; by the universal property of blow-ups ([H1], II, prop. 7.14), there exists a factorization

$$\pi|_{V_1} : V_1 \xrightarrow{\pi_1} X_1 \xrightarrow{\varepsilon_1} U_0 \subset X$$

where the complement of $V_1 = \pi^{-1}(U_0)$ in Y has codimension at least 2 and $\overline{\pi_1(E \cap V_1)}$ is contained in the support of the exceptional divisor of ε_1. If the codimension of $\overline{\pi_1(E \cap V_1)}$ in X_1 is at least 2, the divisor $E \cap V_1$ is contained in the exceptional locus of π_1 and, upon replacing V_1 by the complement V_2 of a closed subset of codimension at least 2 and X_1 by an open subset U_1, we may repeat the construction. After i steps, we get a factorization

$$\pi : V_i \xrightarrow{\pi_i} X_i \xrightarrow{\varepsilon_i} U_{i-1} \subset X_{i-1} \xrightarrow{\varepsilon_{i-1}} \cdots \xrightarrow{\varepsilon_2} U_1 \subset X_1 \xrightarrow{\varepsilon_1} U_0 \subset X$$

as long as the codimension of $\overline{\pi_{i-1}(E \cap V_{i-1})}$ in X_{i-1} is at least 2, where V_i is the complement in Y of a closed subset of codimension at least 2. Let $E_j \subset X_j$ be the exceptional divisor of ε_j. We have

$$\begin{aligned} K_{X_i} &= \varepsilon_i^* K_{U_{i-1}} + c_i E_i \\ &= (\varepsilon_1 \circ \cdots \circ \varepsilon_i)^* K_X + c_i E_i + c_{i-1} E_{i,i-1} + \cdots + c_1 E_{i,1} \end{aligned}$$

where $E_{i,j}$ is the inverse image of E_j in X_i and

$$c_i = \operatorname{codim}_{X_{i-1}}(\overline{\pi_{i-1}(E \cap V_{i-1})}) - 1 > 0$$

([H1], II, Ex. 8.5). Since π_i is birational, $\pi_i^* \mathcal{O}_{X_i}(K_{X_i})$ is a subsheaf of $\mathcal{O}_{V_i}(K_{V_i})$. Moreover, since $\pi_j(E \cap V_j)$ is contained in the support of E_j, the divisor $\pi_j^* E_j - E|_{V_j}$ is effective, hence so is $E_{i,j} - E|_{V_i}$.

It follows that $\mathcal{O}_Y(\pi^* K_X + (c_i + \cdots + c_1)E)|_{V_i}$ is a subsheaf of $\mathcal{O}_{V_i}(K_{V_i}) = \mathcal{O}_Y(K_Y)|_{V_i}$. Since Y is normal and the complement of V_i in Y has codimension at least 2, $\mathcal{O}_Y(\pi^* K_X + (c_i + \cdots + c_1)E)$ is also a subsheaf of $\mathcal{O}_Y(K_Y)$. Since there are no infinite ascending sequences of subsheaves of a coherent sheaf on a Noetherian scheme, the process must terminate at some point: $\overline{\pi_i(E \cap V_i)}$ is a divisor in X_i for some i, hence $E \cap V_i$ is not contained in the exceptional locus of π_i (by 1.40 again). The morphism π_i then induces a birational isomorphism between $E \cap V_i$ and E_i, and the latter is ruled: more precisely, through every point of E_i there is a rational curve contracted by ε_i. This proves the proposition."

page 33

- line 14, for

"The rational map $\pi^{-1} : S \dashrightarrow Y$ might not be a morphism, but its indeterminacies can be resolved by blowing-up a finite number of points"

read

"The indeterminacies of the rational map $\pi^{-1} : S \dashrightarrow Y$ can be resolved by blowing-up a finite number of points"

page 34

- line 15, for

"one may only say by Proposition 1.45 that there is no birational morphism from X_1 to another member of $\mathcal{C}$."

read

"one may only say by Proposition 1.45 that any birational morphism from X_1 to another member of $\mathcal{C}$ is an isomorphism."

page 37

- line 6, for

"a given smooth projective curve C to a given smooth quasi-projective variety."

read

"a fixed projective curve C to a fixed smooth quasi-projective variety."

page 38

- line 8, replace n by N.
- line 15, in equation (2.1), replace u by U and v by V.

page 39

- line 9, for

"Let now X be a subscheme"

read

"Let now X be a (closed) subscheme"

• line 15, for

"of quasi-projective schemes."

read

"of quasi-projective schemes. The same conclusion holds for any quasi-projective variety X."

page 40

• line 2, for

"and a polynomial P"

read

"and a polynomial P with rational coefficients"

• between lines 9 and 10, add a new paragraph:

"The fact that Y is projective is essential in this construction: the space $\mathrm{Mor}(\mathbf{A}^1, \mathbf{A}^N)$ is *not* a disjoint union of quasi-projective varieties."

• line 12, for

$$f^{\mathrm{univ}} : Y \times \mathrm{Mor}(Y, X) \to X \times \mathrm{Mor}(Y, X)$$

such that, for each point t of $\mathrm{Mor}(Y, X)$, the morphism $f_t^{\mathrm{univ}} : Y \to X$ is the morphism"

read

$$f^{\mathrm{univ}} : Y \times \mathrm{Mor}(Y, X) \to X$$

such that, for each point t of $\mathrm{Mor}(Y, X)$, the morphism $f_t^{\mathrm{univ}} : Y \times \{t\} \to X$ is the morphism"

• line 22, for

" • T-morphisms $f : Y \times T \to X \times T$

obtained by sending φ to the pull-back

$$f(y, t) = (pr_1 \circ f^{\mathrm{univ}}(y, \varphi(t)), t)$$

of f^{univ}. Given f (or φ), we will call the morphism

$$\begin{array}{rccc} \mathrm{ev} : & Y \times T & \longrightarrow & X \\ & (y, t) & \longmapsto & f_t(y) = \varphi(t)(y) \end{array}$$

the *evaluation map*."

read

" • morphisms $f : Y \times T \to X$

obtained by sending φ to the pull-back

$$f(y, t) = f^{\mathrm{univ}}(y, \varphi(t))$$

of f^{univ}. Given φ, we will call the morphism f the *evaluation map* and often denote it by ev."

• line -5, for

"the universal morphism being

$$f^{\mathrm{univ}}: \begin{array}{ccc} \{\star\}\times X & \longrightarrow & X\times X \\ (\star,x) & \longmapsto & (x,x) \end{array}$$

Similarly, when Y is a reduced scheme of dimension 0, the scheme $\mathrm{Mor}(Y,X)$ is isomorphic to X^Y. "

read

"the universal morphism being

$$f^{\mathrm{univ}}: \begin{array}{ccc} \{\star\}\times X & \longrightarrow & X \\ (\star,x) & \longmapsto & x \end{array}$$

Similarly, when Y is a reduced scheme of finite length, the scheme $\mathrm{Mor}(Y,X)$ is isomorphic to X^Y. "

page 41

• line 9, for

"which parametrizes rational curves on X whose image is a line"

read

"that parametrizes morphisms $\mathbf{P}^1 \to X$ whose image is a line"

• line 13, for

"$T = \mathrm{Spec}\,\mathbf{k}[\varepsilon]/(\varepsilon^2)$ to $\mathrm{Mor}(Y,X)$ with image $[f]$ ([H1], II, Ex. 2.8), hence extensions of f to T-morphisms

$$f_\varepsilon : Y\times \mathrm{Spec}\,\mathbf{k}[\varepsilon]/(\varepsilon^2) \to X\times \mathrm{Spec}\,\mathbf{k}[\varepsilon]/(\varepsilon^2)$$

read

"$\mathrm{Spec}\,\mathbf{k}[\varepsilon]/(\varepsilon^2)$ to $\mathrm{Mor}(Y,X)$ with image $[f]$ ([H1], II, Ex. 2.8), hence extensions of f to morphisms

$$f_\varepsilon : Y\times \mathrm{Spec}\,\mathbf{k}[\varepsilon]/(\varepsilon^2) \to X$$

page 42

• line 14, for

"**Theorem 2.6** *Let X and Y be projective varieties and let $f : Y \to X$ be a morphism such that X is smooth along $f(Y)$.*"

read

"**Theorem 2.6** *Let X be a quasi-projective variety, let Y be a projective variety, and let $f : Y \to X$ be a morphism such that X is smooth along $f(Y)$.*"

• line -5, for

" The canonical morphism $\mathrm{Spec}(R/I) \to \mathrm{Mor}(Y,X)$ corresponds to an extension $f_{R/I} : Y\times \mathrm{Spec}(R/I) \to X\times \mathrm{Spec}(R/I)$ of f. Since $I^2 \subset \mathfrak{m}I$, the obstruction to extending it to a morphism $f_{R/\mathfrak{m}I} : Y\times \mathrm{Spec}(R/\mathfrak{m}I) \to X\times \mathrm{Spec}(R/\mathfrak{m}I)$ lies by Lemma 2.7 below in"

read

" The canonical morphism $\mathrm{Spec}(R/I) \to \mathrm{Mor}(Y,X)$ corresponds to an extension $f_{R/I} : Y\times \mathrm{Spec}(R/I) \to X$ of f. Since $I^2 \subset \mathfrak{m}I$, the obstruction to extending it to a morphism $f_{R/\mathfrak{m}I} : Y\times \mathrm{Spec}(R/\mathfrak{m}I) \to X$ lies by Lemma 2.7 below in"

page 43

- line 13, for

"Let R be a finitely generated local $\mathbf{k}$-algebra"

read

"Let R be a Noetherian local $\mathbf{k}$-algebra"

- line 15, for

$$f_{R/I} : Y \times \mathrm{Spec}(R/I) \to X \times \mathrm{Spec}(R/I)$$

be an extension of f. Assume X is smooth along the image of f. The obstruction to extending $f_{R/I}$ to a morphism

$$f_R : Y \times \mathrm{Spec}(R) \to X \times \mathrm{Spec}(R)$$

read

$$f_{R/I} : Y \times \mathrm{Spec}(R/I) \to X$$

be an extension of f. Assume X is smooth along the image of f. The obstruction to extending $f_{R/I}$ to a morphism

$$f_R : Y \times \mathrm{Spec}(R) \to X$$

page 44

- line 2, for

"([H1], II, prop. 8.2A)"

read

"([Bo], X, § 7, n° 10, déf. 2)"

- line 16, for

"is necessary and sufficient for a global extension to exist"

read

"is necessary and sufficient for a global extension to exist (on a separated Noetherian scheme, the cohomology of a coherent sheaf is isomorphic to its Čech cohomology relative to any open affine covering; [H1], III, Theorem 4.5)."

- line 22, for

"Let σ be a section of π. If a and b are in $\mathfrak{m}$, we have $\sigma \circ \pi(a) = a + a'$ and $\sigma \circ \pi(b) = b + b'$, where a' and b' are in I. Then"

read

"Let σ be a section of π: if a and b are in R, we can write $\sigma \circ \pi(a) = a + a'$ and $\sigma \circ \pi(b) = b + b'$, where a' and b' are in I. If a and b are in $\mathfrak{m}$, we have"

page 45

- line 3, for

"We will need a slightly more general situation:"

read

“Again, X and Y are varieties over a field $\mathbf{k}$, with Y projective and X quasi-projective. We will need to study a slightly more general situation:”

• line 12, for

“Note that when B is finite,”

read

“Note that when B is finite and X is smooth,”

• line 16, for

“locally at a point $[f]$ such that X is smooth along $f(Y)$, *the scheme* $\mathrm{Mor}(Y, X; g)$ *can be defined*”

read

“around a point $[f]$ such that X is smooth along $f(Y)$, *the scheme* $\mathrm{Mor}(Y, X; g)$ *can be locally defined*”

• line -7, for

“Y is a curve C (and B is finite):”

read

“Y is a curve C (as usual, reduced connected proper over a field) and B is finite:”

page 46

• line 3, for

“over an irreducible base S”

read

“over an irreducible scheme S of finite type over a field”

• line 4, for

“and a morphism”

read

“, a point s of S, and a $k(s)$-morphism”

• line -4 (footnote), for the references

“(1.7.1), (1.7.2), (1.7.3)”

read

“II.(1.7.1), II.(1.7.2), II.(1.7.3)”

page 48

• line 14, in formula (2.5), for

“$\mathcal{O}_\ell(1)^{n-1}$”

read

“$\mathcal{O}_\ell(1)^{N-1}$”

page 49

• line 3, for

"The exact sequence (2.5) yields"

read

"When X is smooth along ℓ (which certainly happens if the equations defining X are general), the exact sequence (2.5) yields"

• line 9, for

"$\alpha(\lambda_2, \ldots, \lambda_n)$"

read

"$\alpha(\lambda_2, \ldots, \lambda_N)$"

page 50

• line 9, for

$$\sum_{i \in I_j} x_i^d = 0 \qquad \text{for all } j$$

read

$$x_{I_j} \neq 0 \quad \text{and} \quad \sum_{i \in I_j} x_i^d = 0 \qquad \text{for all } j$$

page 51

• line -1, for

"and the characteristic of $\mathbf{k}$ is $\geq d$."

read

"and the characteristic of $\mathbf{k}$ is 0 or $\geq d$."

page 53

Replace the beginning of Exercise 3 by

3. (a) If p is positive and $d-1$ is a power of p, show that there are exactly $d^3(d-3)$ nonstandard lines on the surface X_3^d.

Throughout the rest of this exercise, we assume $p = 0$ or $p > d$.

(a') Assume $d \geq 3$. Show that all lines on X_3^d are standard (see 2.14). There are therefore $3d^2$ of them.

(b) Let ℓ be a standard line contained in X_N^d which is generic in its $\mathbf{P}^{r-1}$ (with the notation of 2.14). Show

$$h^0(\ell, N_{\ell/X_N^d}(-1)) = N - 1 - \min(r, d) \qquad h^0(\ell, N_{\ell/X_N^d}) = 2N - 2 - \min(2r, d+1)$$

page 54

In Exercise 3.c), add a footnote:

"It was shown by Green (*Some Picard theorems for holomorphic maps to algebraic varieties*, Amer. J. Math. **97** (1975), 43–75) that over the complex numbers, for $d > n^2 - 1$, the image of any *holomorphic* map $\mathbf{C}^m \to X_n^d$ is contained in a linear subspace contained in X_n^d of the type described page 64. In particular, any rational or elliptic curve in X_n^d, or more generally, any image in X_n^d of a projective space or a complex torus, is contained in a linear subspace of this form."

page 56

• line -9, for

"1-cycle $\sum_{i=1}^r (\deg f|_{C_i}) f(C_i)$."

read

"1-cycle $\sum_{i=1}^r d_i f(C_i)$, where d_i is the degree of $f|_{C_i}$ onto its image. Note that for any Cartier divisor D on X, one has $D \cdot f_*C = \deg(f^*D)$."

page 57

• line 5, delete

"connected"

• line 8, delete

"We may assume that C is irrational"

• After the illustration, read

" *The* 1*-cycle* f_*C *degenerates to a* 1*-cycle with a rational component* $e(E)$*.*

If ev is defined at every point of $\{c\} \times \overline{T}$, the rigidity lemma 1.15(a) implies that there exist a neighborhood V of c in C and a factorization

$$\mathrm{ev}|_{V \times \overline{T}} : V \times \overline{T} \xrightarrow{p_1} V \xrightarrow{g} X$$

The morphism g must be equal to $f|_V$. It follows that ev and $f \circ p_1$ coincide on $V \times T$, hence on $C \times T$. But this means that the image of T in $\mathrm{Mor}(C, X; f|_{\{c\}})$ is just the point $[f]$, and this is absurd.

Hence there exists a point..."

page 59

• After the illustration, for

" *The rational* 1*-cycle* $f(C)$ *bends and breaks.* "

read

" *The* 1*-cycle* f_*C *bends and breaks.*

• the text from the last paragraph of the page to the end of the proof should read:

" Since $\overline{T}$ is a smooth curve and S is integral, π is flat ([H1], III, prop. 9.7). Assume that its fibers are all integral. Their genus is then constant ([H1], III, cor. 9.10), hence equal to 0. Therefore, each fiber is a smooth rational curve.

The surface S is then a (minimal) ruled surface in the sense of [H1], V, §2 (Hartshorne assumes that S is smooth, but this hypothesis is not used in the proofs hence follows from the others). Let T_0 be the closure of $\{0\} \times T$ in S and let T_∞ be the closure of $\{\infty\} \times T$. These sections of π are contracted by e (to $f(0)$ and $f(\infty)$ respectively).

If H is an ample divisor on $e(S)$, which is a surface by construction, we have $(e^*H)^2 > 0$ and $e^*H \cdot T_0 = e^*H \cdot T_\infty = 0$, hence T_0^2 and T_∞^2 are negative by the Hodge index theorem.

However, since T_0 and T_∞ are both sections of π, their difference is linearly equivalent to the pull-back by π of a divisor on $\overline{T}$ ([H1], V, prop. 2.3). In particular,

$$0 = (T_0 - T_\infty)^2 = T_0^2 + T_\infty^2 - 2T_0 \cdot T_\infty < 0$$

which is absurd.

It follows that at least one fiber F of π is not integral. Since S is normal, the fibers have no embedded points, hence F is either reducible or has a multiple component. By Lemma 3.7, the components of F are all rational curves, and they are not contracted by e. The direct image of F on X is the required 1-cycle."

• In footnote 1, add:

" Alternatively, the construction can be made as follows. The indeterminacies of the rational map $\mathbf{P}^1 \times \overline{T} \dashrightarrow X \times \overline{T}$ induced by F can be resolved by blowing up points to get a morphism $\overline{F}' : S' \to \mathbf{P}^1 \times \overline{T} \dashrightarrow X \times \overline{T}$ whose Stein factorization is $\overline{F}' : S' \to S \xrightarrow{\overline{F}} X \times \overline{T}$ In other words, the surface S is obtained from S' by contracting the components of fibers of $S' \to \overline{T}$ that are contracted by $\overline{F}'$."

page 60

• line -7, for

"if D is a divisor associated with the line bundle $\mathcal{O}_Y(1)$"

read

"if D is a divisor associated with the line bundle $\mathcal{O}_X(1)$"

• line -5, for

$$-K_X = (r+1)D + \pi^*(-K_Y - D_1 - \cdots - D_r)$$

read

$$-K_X = rD + \pi^*(-K_Y - D_1 - \cdots - D_r)$$

page 61

• line 2, for

"the relative dualizing sheaf $\omega_{\mathcal{X}_U/U}$ is ample"

read

"the dual $\omega^*_{\mathcal{X}_U/U}$ of the relative dualizing sheaf is ample"

page 62

• line 25, for

"It follows that $-K_Y$ is ample"

read

"It follows that $-K_X$ is ample"

page 64

• line 7, for

$$H \cdot \Gamma \le \frac{2H \cdot C}{\mathrm{Card}(B)}$$

read

$$H \cdot \Gamma \leq \frac{2H \cdot f_* C}{\mathrm{Card}(B)}$$

page 65

• After the illustration, for

“ *The curve $f(C)$ bends and breaks keeping $c_1, \dots, c_b$ fixed.* ”

read

“ *The 1-cycle $f_* C$ bends and breaks, keeping $c_1, \dots, c_b$ fixed.*

• line -5, for

“$N^1(X)_{\mathbf{R}}$”

read

“$N^1(S)_{\mathbf{R}}$”

page 67

• line 11, for

$$H \cdot \Gamma \leq 2 \dim(X) \, \frac{H \cdot C}{-K_X \cdot C}$$

read

$$H \cdot \Gamma \leq 2 \dim(X) \, \frac{H \cdot f_* C}{-K_X \cdot f_* C}$$

• lines -7 and -5, for

$$-p^m K_X \cdot C$$

read

$$-p^m K_X \cdot f_* C$$

• line -3, for

“a rational curve Γ_m”

read

“a rational curve Γ_m on X”

• line -1, for

$$H \cdot \Gamma_m \leq \frac{2H \cdot C_m}{b_m} = \frac{2p^m}{b_m}(H \cdot C)$$

read

$$H \cdot \Gamma_m \leq \frac{2H \cdot (f_m)_* C_m}{b_m} = \frac{2p^m}{b_m} \; H \cdot f_* C$$

page 68

- line 1, for

"$-\dim(X)/(-K_X \cdot C)$."

read

"$-\dim(X)/(-K_X \cdot f_*C)$."

- line 3, for

$$H \cdot \Gamma_m \leq \frac{2\dim(X)}{-K_X \cdot C}(H \cdot C)$$

read

$$H \cdot \Gamma_m \leq \frac{2\dim(X)}{-K_X \cdot f_*C}\ H \cdot f_*C$$

- line 13, for

"of degree at most $2\dim(X)\frac{H \cdot C}{-K_X \cdot C}$"

read

"of degree at most $2\dim(X)\frac{H \cdot f_*C}{-K_X \cdot f_*C}$"

- line 17, for

"Let M_d be the quasi-projective scheme that parametrizes rational curves on X"

read

"Let M_d be the quasi-projective scheme that parametrizes morphisms $\mathbf{P}^1 \to X$"

page 71

- line 1, delete

"connected"

- After the illustration, for

" *The curve $f(C)$ bends and breaks: ...* "

read

" *The 1-cycle f_*C bends and breaks: ...*

page 77

- line 2, for

"any such morphism factors through α_X."

read

"any such morphism factors uniquely through α_X."

page 79

- line -4, for

"$\pi^{-1}(\pi(x))$"

read

“$\pi^{-1}(\pi(f(c)))$”

page 80

• After the illustration, for

“ *The curve $f(C)$ bends and breaks: a non vertical rational 1-cycle $e(E)$ appears.* ”

read

“ *The 1-cycle f_*C bends and breaks: a nonvertical 1-cycle e_*E appears.*

page 81

• line -12, delete

“connected”

page 83

• line -10, delete

“connected”

page 88

• line 4, for

“It follows that the map ev_d^K is also dominant.”

read

“It follows that the map ev_d^k is also dominant.”

• line 14, for

“the morphism $\mathrm{Mor}_d(\mathbf{P}^1, \tilde{X}) \to \mathrm{Mor}_d(\mathbf{P}^1, X)$”

read

“the morphism $\mathrm{Mor}(\mathbf{P}^1, \tilde{X}) \to \mathrm{Mor}(\mathbf{P}^1, X)$”

page 89

• lines 1 and 16, for

“for some d”

read

“for some positive d”

• line 17, for

“the morphism $\mathrm{Mor}_d(\mathbf{P}^1, \tilde{X}) \to \mathrm{Mor}_d(\mathbf{P}^1, X)$”

read

“the morphism $\mathrm{Mor}(\mathbf{P}^1, \tilde{X}) \to \mathrm{Mor}(\mathbf{P}^1, X)$”

page 90

• line 12, for

“ With our notation, this means $a_n \geq r$. We will say”

read

"   With our notation, this means $a_n \geq r$. Note that the composition of an r-free morphism with any automorphism of $\mathbf{P}^1$ is still r-free. We will say that a rational curve on X is r-free if its normalization is.

We will say"

- line 22, for

"by Theorem 2.6."

read

"by Theorem 2.6. and its extension 2.9."

- line -8, for

"there is an exact sequence"

read

"there is an exact sequence ([H1], II, Theorem 8.17)"

page 93

- line -12, delete

"Also, the cyclic cover X of degree p of $\mathbf{P}^n$, with $n \geq p$, branched along a general hypersurface of degree p is a Fano (hence uniruled) variety which has no free rational curves ([K1], th. V.5.11)."

page 94

- line -7, for

"Let $\mathrm{ev}_i : \mathbf{P}^1 \times M_i \to X$ be the evaluation maps."

read

"Let $e_i : \mathbf{P}^1 \times (M_i)_{\mathrm{red}} \to X$ be the morphisms induced by the evaluation maps."

and replace ev_i by e_i in the rest of the proof.

- line -3, for

"[H1], III, prop. 4.6"

read

"[H1], III, Corollary 10.7"

page 97

- line 15, for

"[H1], III, prop. 4.6"

read

"[H1], III, Corollary 10.7"

page 98

- line 11, for

"the injection $\iota : \{0\} \times M \hookrightarrow X$"

read

"the injection $\iota : \{0\} \times M \hookrightarrow \mathbf{P}^1 \times M$"

- line 13, for

$$\pi_1(\iota) \circ \pi_1(\mathrm{ev}) = 0$$

read

$$\pi_1(\mathrm{ev}) \circ \pi_1(\iota) = 0$$

- line -15, for

"Let $\mathrm{ev}_i : \mathbf{P}^1 \times M_i \to X$ be the evaluation maps."

read

"Let $e_i : \mathbf{P}^1 \times (M_i)_{\mathrm{red}} \to X$ be the morphisms induced by the evaluation maps."

and replace ev_i by e_i in the rest of the proof.

- line -10, for

"X^{free}"

read

"X_x^{free}"

page 100

- in footnote 4, for

"The proof follows the arguments used for uniruled or rationally connected varieties, using the closed subscheme $\mathrm{Rat}(X)$ of $\mathrm{Hilb}(X)$ that will be defined in 5.6, instead of $\mathrm{Mor}(\mathbf{P}^1, X)$."

read

"When X is projective, the proof follows the arguments used for uniruled or rationally connected varieties, using the closed subscheme $\mathrm{Rat}(X)$ of $\mathrm{Hilb}(X)$ that will be defined in 5.6, instead of $\mathrm{Mor}(\mathbf{P}^1, X)$. For the general case, see [K1], prop. IV.3.6."

page 114

- line -15, for

"an irreducible constructible subset"

read

"a constructible subset"

- from line -12 on, read

"The right-hand side is contained in the left-hand side. Let us prove the other inclusion. We may assume by Lemma 5.1 that V is irreducible. By [H1], II, ex. 3.18(b), V contains a subset U that is dense open in $\overline{V}$. Let W be a component of $\overline{V} - U$. By a theorem of Chevalley ([H1], II, ex. 3.22(e)), a general fiber of $W \to \pi(\overline{V})$ is either empty or everywhere of dimension

$$\dim(W) - \dim(\pi(\overline{V}))$$

which is less than the dimension $\dim(\overline{V}) - \dim(\pi(\overline{V}))$ of the fiber of $\overline{V} \to \pi(\overline{V})$ at the same point. It follows that for y general in $\pi(V)$, the closed subset $(\overline{V} - U) \cap \pi^{-1}(y)$ of $\overline{V} \cap \pi^{-1}(y)$ is nowhere dense, hence that $U \cap \pi^{-1}(y)$ is dense in $\overline{V} \cap \pi^{-1}(y)$."

page 116

- line 8, for

"separable closure of K and let"

read

"separable closure of K, and let"

page 117

- line -5, for

"of length m"

read

"of length 1"

page 118

- line 5, for

"$F(\overline{W_m^{i,j}})$"

read

"$\overline{F(W_m^{i,j})}$"

- line -12 on, read

" Call $\overline{V_m^i}$ *stable* if $\overline{F(\tilde{W}_m^{i,j})} = \overline{V_m^i}$ for all j, *unstable* otherwise. Note the following:

- if all components of $\overline{V_m(x)}$ are stable, $\overline{V_{m+1}(x)} = \overline{V_m(x)}$;
- if $\overline{V_m^i}$ is unstable, it is strictly contained in an irreducible component of $\overline{V_{m+1}(x)}$;
- if $\overline{V_m^i}$ is stable and an irreducible component of $\overline{V_{m+1}(x)}$, it is stable as a component of $\overline{V_{m+1}(x)}$.

Let $\overline{V_m^0} = \overline{F(\tilde{W}_{m-1}^{i,j})}$ be an unstable component of $\overline{V_m(x)}$. If the corresponding $\overline{V_{m-1}^i}$ is a stable component of $\overline{V_{m-1}(x)}$, it is equal to $\overline{V_m^0}$ by definition, which contradicts the third item. Hence $\overline{V_{m-1}^i}$ is unstable, and is strictly contained in $\overline{V_m^0}$ by the second item.

It follows that if $\overline{V_m(x)}$ has an unstable component $\overline{V_m^0}$, then $\overline{V_{m-1}(x)}$ also has an unstable component of smaller dimension. In particular, the dimension of $\overline{V_m^0}$ is at least m. By the second item, this implies $\dim(\overline{V_{m+1}(x)}) > m$, which is impossible since $m \geq \delta(x)$. Therefore, $\overline{V_m(x)}$ has only stable components, and $\overline{V_{m+1}(x)} = \overline{V_m(x)}$ by the first item. This proves the first step."

page 119

- line 5, for

$$V = \bigcup_{x \in X} \big(\{x\} \times V_n(x)\big)$$

read

$$V = \bigcup_{x \in X} \big(V_n(x) \times \{x\}\big)$$

page 123

- line -2, for

"hence factors as $X'^* \to X^* \xrightarrow{\rho} Y$,"

read

"hence factors as $X'^* \to X^* \xrightarrow{\rho} Y^*$,"

page 131

- line -15, for

"Let $\mathrm{Rat}_{n+1}(X)$ be the closure in $\mathrm{Hilb}(X)$ of the scheme that parametrizes curves"

read

"Let $\mathrm{Rat}_{n+1}(X)$ be the (projective) subscheme of $\mathrm{Hilb}(X)$ that parametrizes curves"

page 133

- line 7, delete

"Examples in Section 5.11 show that k may grow as $\log n$."

page 135

- line -9, for

"8 points"

read

"8 points in general position"

- line -6, for

"87 other families with Picard number > 1 ([MM])."

read

"88 other families with Picard number > 1 (Mori and Mukai recently noticed that the family of blow-ups of $\mathbf{P}^1 \times \mathbf{P}^1 \times \mathbf{P}^1$ along a curve of tridegree $(1, 1, 3)$ is missing from the list in [MM])."

page 136

- line -18, for

"the curve $C_1 = f^{-1}(C)$."

read

"the curve $C_1 = \pi^{-1}(C)$."

pages 136 and 137

- delete footnote 15 and replace footnote 16 with

" In this direction, complex **Q**-Fano threefolds with canonical singularities (see Definition 7.13) and Picard number 1 are known to form a limited family ([Ka4] for the case of terminal singularities; the canonical case is in Kollár, J., Miyaoka, Y., Mori, S. and Takagi, H., Boundedness of canonical **Q**-Fano 3-folds, *Proc. Japaan Acad. Ser. A Math. Sci.* **76** (2000), 73–77). The point is again to bound $(-K_X)^n$ *and* an integer j such that jK_X is a Cartier divisor. Work of Ran and Clemens ([RC]) shows that **Q**-Fano n-folds with canonical singularities and Picard number 1 form a limited family if one moreover bounds the smallest number m such that $-mK_X$ is very ample (in the smooth case, one can take $m = n(n+1)(n+3)$ as in the proof of Theorem 5.19).

More generally, it is conjectured that given $\varepsilon > 0$, Fano varieties X such that, for any desingularization $f : \tilde{X} \to X$, one has $K_{\tilde{X}} = f^*K_X + \sum a_i E_i$ with $a_i > -1 + \varepsilon$, where the E_i are the exceptional divisors of f, still form a limited family. Our surfaces fall in this class for $g = 0$ and $d < 2/\varepsilon$."

page 138

- line 2, read,

"or by noting that L and H are nef, and $L + H$ is ample, since its associated line bundle is the tautological line bundle associated with the description of X as $\mathbf{P}(\mathcal{O}^r_{\mathbf{P}^s}(1) \oplus \mathcal{O}^r_{\mathbf{P}^s}(a+1))$; it is therefore ample)."

- replace lines 11 through 15 with

Setting $n = \dim(X) = r + s$, we get, since L and H are nef, for $a = s$

$$(-K_X)^n \geq ((r+1)L)^n = (r+1)^n a^s L^r \cdot H^s = (r+1)^n a^s$$

- line -1, for

$H^0(\mathbf{P}^s, \mathcal{O}_{\mathbf{P}^s}(s-r))^r$

read,

$H^0(\mathbf{P}^s, \mathcal{O}_{\mathbf{P}^s}(s))^r$

page 141

- line 18, for

"yields the (5.3)."

read

"yields (5.3)."

page 146

- line 10, for

" the Riemann-Roch theorem yields

$$h^0(X, \mathcal{O}_X(mC)) = m + \chi(X, \mathcal{O}_X) \geq 2$$

and the linear system $|mC|$ has no base-point (the only possible fixed curve is C, but $h^0(X, \mathcal{O}_X((m-1)C)) < h^0(X, \mathcal{O}_X(mC))$, and there are no isolated base-points since $C^2 = 0$)."

read

" the Riemann-Roch theorem yields

$$h^0(X, \mathcal{O}_X(mC)) \geq m + \chi(X, \mathcal{O}_X) \geq 2$$

Since the only possible fixed curve in the linear system $|mC|$ is C, the moving part of the linear system $|mC|$ is of the form $|m'C|$ for some nonnegative integer $m' \leq m$. Since $(m'C)^2 = 0$, the linear system $|m'C|$ has no base-points."

page 147

- line -2, for

"Lemma 6.2(d)"

read

"Lemma 6.2(e)"

page 149

- l.14, for

"and E the last (-1)-curve of the composition of blow-ups ε, its image in X is a curve,"

read

"and if E is the last (-1)-curve of the composition of blow-ups ε, its image $\tilde{\sigma}(E)$ in X is a curve,"

- l.18, for

"there are no exceptional curves in $\tilde{X}$,"

read

"there are no such curves E in $\tilde{X}$,"

page 152

- line 3, for

$$H \cdot \Gamma_i < \varepsilon^{-1} K_X \cdot \Gamma_i \leq \varepsilon^{-1}(\dim(X) + 1)$$

read

$$H \cdot \Gamma_i < -\frac{1}{\varepsilon}\ K_X \cdot \Gamma_i \leq \frac{1}{\varepsilon}\ (\dim(X) + 1)$$

- line -5, starting from "Let V_J be this cone..." replace the end of the proof with

" By Lemma 6.7(b), it is enough to show that any extremal ray $\mathbf{R}^+ r$ in $\overline{V_J}$ satisfying $K_X \cdot r < 0$ is in V_J. Let H be an ample divisor on X and let ε be a positive number such that $(K_X + \varepsilon H) \cdot r < 0$. By the first step, there are only finitely many classes $z_{j_1}, \ldots, z_{j_q}$, with $j_\alpha \in J$, such that $(K_X + \varepsilon H) \cdot z_{j_\alpha} < 0$.

Write r as the limit of a sequence $(r_m + s_m)$, where $r_m \in \overline{\mathrm{NE}}(X)_{K_X + \varepsilon H \geq 0}$ and $s_m = \sum_{\alpha=1}^{q} l_{\alpha,m} z_{j_\alpha}$. Since $H \cdot r_m$ and $H \cdot z_{j_\alpha}$ are positive, the sequences $(H \cdot r_m)$ and $(l_{\alpha,m})$ are bounded, hence we may assume, after taking subsequences, that all sequences (r_m) and $(l_{\alpha,m})$ have limits (Theorem 1.27(b)). Because r spans an extremal ray in $\overline{V_J}$, the limits must be nonnegative multiples of r, and since $(K_X + \varepsilon H) \cdot r < 0$, the limit of (r_m) must vanish. Moreover, r is a multiple of one the z_{j_α}, hence is in V_J.

If we choose a set I of indices such that $(\mathbf{R}^+ z_j)_{j \in I}$ is the set of all (distinct) extremal rays among all $\mathbf{R}^+ z_i$, the proof shows that any extremal ray of $\overline{\mathrm{NE}}(X)_{K_X < 0}$ is spanned by a z_i, with $i \in I$. This finishes the proof of the cone theorem. □"

page 154

- line -14, for

"component of R"

read

"component of the locus of R"

page 157

• lines 31–32, read

"

$$K_X = -r\xi + \pi^*(K_Y + \det(\mathcal{E}))$$

If ℓ is the class of a line contained in a fiber of π, we have $K_X \cdot \ell = -r$."

page 158

• lines 19 and 24, for

"$d > 2g - 1$"

read

"$d \geq 2g - 1$"

page 160

• footnote 5: line -7, for

"Its blow-up is the total space of the $\mathbf{P}^1$-bundle $\mathbf{P}(\mathcal{O}_Q \oplus \mathcal{O}_Q(1)) \to Q$, where Q is a smooth quadric in $\mathbf{P}^3$, and the exceptional divisor is the image of the section corresponding to the trivial quotient of $\mathcal{O}_Q \oplus \mathcal{O}_Q(1)$."

read

"If Q is a smooth quadric in $\mathbf{P}^3$, and Q_0 and Q_∞ are the two sections of the $\mathbf{P}^1$-bundle $\mathbf{P}(\mathcal{O}_Q \oplus \mathcal{O}_Q(1)) \to Q$ corresponding to the quotients $\mathcal{O}_Q$ and $\mathcal{O}_Q(1)$ of $\mathcal{O}_Q \oplus \mathcal{O}_Q(1)$, its blow-up is $\mathbf{P}(\mathcal{O}_Q \oplus \mathcal{O}_Q(1)) \boldsymbol{-} Q_\infty$ and the exceptional divisor is Q_0."

page 161

• footnote 7: line -6, read

$$\left(1 - a_{22} - \frac{a_{12}a_{21}}{1 - a_{11}}\right)[L_2] = \frac{a_{21}}{1 - a_{11}}(r_1 + z_1) + (r_2 + z_2)$$

page 162

• line 30, read

$$S_1^+ = \mathbf{P}(N^*_{\Gamma^+/X^+}) = \mathbf{P}(\mathcal{O}_{\mathbf{P}^1} \oplus \mathcal{O}_{\mathbf{P}^1}(1))$$

• footnote 9: line -3, for

"with exceptional divisor $S = \mathbf{P}(N_{C/Y})$."

read

"with exceptional divisor $S = \mathbf{P}(N^*_{C/Y})$."

page 163

• line 13, for

"relation $0 = -a + b$, which is absurd."

read

"relation $-1 = -a + b$, which is absurd."

- footnote 9: line -1, for

"which is split if $\deg(N_{C/Y}) \geq 3C_0^2 - 2$."

read

"which is split if $\deg(N_{C/Y}) \leq 3C_0^2 + 2$."

page 165

- line 8, for

"let M be a nef divisor"

read

"let M be an ample divisor"

page 178

- line 3, for

"c) for some desingularization $\pi : Y \to X$, any j-canonical form on X_{reg} extends to an j-canonical form on Y."

read (the stronger, local condition)

"c) for some desingularization $\pi : Y \to X$ and any open subset $U \subset X$, any j-canonical form on U_{reg} extends to a j-canonical form on $\pi^{-1}(U)$."

page 179

- line 1, for

"with poles at every exceptional divisor"

read

"that vanishes along every exceptional divisor"

page 181

- line 16, for

"let $-D$ be an"

read

"let $-D$ be a"

page 184

- footnote 8: for

"There are now simple proofs of this statement (see [AJ], [BP])."

read

"There are now simple proofs of this statement (see [AJ], [BP], and especially K. H. Paranjape, *The Bogomolov–Pantev resolution, an expository account*, New trends in algebraic geometry (Warwick, 1996), Cambridge University Press, Cambridge, 1999, pp. 347–358)."

page 189

• line 15, for

$$H^0(X, m\pi^*D + E)$$

read

$$H^0(Y, m\pi^*D + E)$$

• line 21, for

$$K_Y + \pi^*A$$

read

$$K_Y$$

page 191

• Invoking Riemann-Roch in the first step of the proof of the theorem is a bit overkill. The following more elementary statement, more in the spirit of Chapter 1 of this book, is sufficient for our purpose here. Its proof can be found in [Kle], §2, Theorem 1, or Theorem 9.6.3 of the more recent reference: S. Kleiman, The Picard scheme. *Fundamental algebraic geometry,* 235–321, Math. Surveys Monogr. **123**, Amer. Math. Soc., Providence, RI, 2005.

Let X a proper scheme of dimension n over a field and let D be a Cartier divisor on X. The following properties are equivalent:

(i) *the divisor D is numerically equivalent to 0;*

(ii) *for any coherent sheaf $\mathcal{F}$ on X, we have $\chi(X, \mathcal{F}(D)) = \chi(X, \mathcal{F})$;*

(iii) *for all Cartier divisors $D_1, \dots, D_{n-1}$ on X, we have $(D \cdot D_1 \cdot \ldots \cdot D_{n-1}) = 0$;*

(iv) *for any Cartier divisor E on X, we have $(D \cdot E^{n-1}) = 0$.*

(The only difficult implication is (i) ⇒ (ii).)

page 193

• line −3, delete

“and $b_{i_0} > -1$”

page 195

• line 25, for

“let $B(b)$ be the base-locus”

read

“let $B(b)$ be the (reduced) base-locus”

page 196

• lines 9–12, read

“(b) $K_Y \sim \pi^*(K_X + \Delta) + \sum_i a_i F_i$, with $a_i > -1$ for all i;

(c) the divisor $\pi^*(aD - (K_X + \Delta)) - \sum_i p_i F_i$ is ample, where $p_i \in (0, 1 + a_i)$ for all i.”

- line -10, for “property (b)”, read “property (c)”
- line -3, for “$N_{b,c}$”, read “$N_{m,c}$”

page 197

- line -7, for “rays).” read “rays.”
- line -1, for “(1.29).” read “(1.29)).”

page 199

- line -14, for

“Let $B(p,q)$ be its base-locus.”

read

“Let $B(p,q)$ be its (reduced) base-locus.”

page 200

- line -9 on, read

“There exists as in the proof of the base-point-free theorem 7.32 a desingularization $\pi : Y \to X$ and divisors F_i on Y (not necessarily exceptional) such that

(a) the linear system $|\pi^*(p_0 H + q_0 j_{X,\Delta}(K_X + \Delta)) - \sum_i r_i F_i|$ is base-point-free, where $r_i \geq 0$ for all i;

(b) $K_Y \sim \pi^*(K_X + \Delta) + \sum_i a_i F_i$, with $a_i > -1$ for all i;

(c) the divisor $\pi^*(p_0 H + (q_0 j_{X,\Delta} - 1)(K_X + \Delta)) - \sum_i p_i F_i$ is ample, where $p_i \in (0, 1 + a_i)$ for all i.”

page 201

- line 13, for “aq_0” read “$j_{X,\Delta} q_0$”
- line -11, for “$-(K_X + \Delta)$” read “$-\pi^*(K_X + \Delta)$”

page 202

- line 10, for

“the base-locus of the left-hand side”

read

“the (reduced) base-locus of the left-hand side”

page 203

- line 26, for

$$\frac{u_m}{v} = \sup\{t \in \mathbf{R} \mid mM + H + t(K_X + \Delta) \text{ nef}\}$$

read

$$\frac{j_{X,\Delta} u_m}{v} = \sup\{t \in \mathbf{R} \mid mM + H + t(K_X + \Delta) \text{ nef}\}$$

- line 29, for

"$(mM+H+\frac{u_m}{v}(K_X+\Delta)\cdot z_0 \geq 0$ "

read

"$(mM+H+\frac{j_{X,\Delta}u_m}{v}(K_X+\Delta)\cdot z_0 \geq 0$ "

- line 30, for

$$u_m \leq v\ \frac{(mM+H)\cdot z_0}{-(K_X+\Delta)\cdot z_0} = v\ \frac{H\cdot z_0}{-(K_X+\Delta)\cdot z_0}$$

read

$$j_{X,\Delta}u_m \leq v\ \frac{(mM+H)\cdot z_0}{-(K_X+\Delta)\cdot z_0} = v\ \frac{H\cdot z_0}{-(K_X+\Delta)\cdot z_0}$$

- line 33, for

$$L_m = v(mM+H+\frac{u_\infty}{v}(K_X+\Delta))$$

read

$$L_m = v(mM+H+\frac{j_{X,\Delta}u_\infty}{v}(K_X+\Delta))$$

page 204

- line 8, for

$$(L_H)|_{V_M} = (H+\frac{u_\infty}{v}(K_X+\Delta))|_{V_M}$$

read

$$(L_H)|_{V_M} = (H+\frac{j_{X,\Delta}u_\infty}{v}(K_X+\Delta))|_{V_M}$$

- line 11, for

"*Second step:* $\overline{\mathrm{NE}}(X)$ *is the closure of*

$$V = \overline{\mathrm{NE}}(X)_{K_X+\Delta\geq 0} + \sum_{L\in\mathcal{L}} V_L$$

where $\mathcal{L}$ *is the set of nef nonample divisors* L *such that* V_L *is a* $(K_X+\Delta)$*-negative extremal ray.*

By Lemma 6.7(a), the boundary of the dual cone $\overline{\mathrm{NE}}(X)^*$ meets the interior of $\overline{V}^*$ at a point that corresponds to a nonample nef $\mathbf{R}$-divisor M positive on $\overline{V} - \{0\}$."

read

"*Second step:* $\overline{\mathrm{NE}}(X)$ *is the closure of*

$$V = \overline{\mathrm{NE}}(X)_{K_X+\Delta\geq 0} + \sum_{V_L\in\mathcal{L}} V_L$$

where $\mathcal{L}$ is the set of $(K_X + \Delta)$-negative extremal rays of the type V_L, with L nef nonample divisor.

If this is not the case, there exists by Lemma 6.7(d) (since $\overline{\mathrm{NE}}(X)$ contains no lines) an $\mathbf{R}$-divisor M on X which is nonnegative on $\overline{\mathrm{NE}}(X)$ but vanishes on some extremal ray (it is in particular nef and nonample), and is positive on $\overline{V} - \{0\}$."

• line -4, for

"*Third step: $\mathcal{L}$ is countable and the $(K_X + \Delta)$-negative rays $(V_L)_{L\in\mathcal{L}}$ are locally discrete in the half-space $N_1(X)_{K_X+\Delta<0}$.*"

read

"*Third step: $\mathcal{L}$ is countable and locally discrete in the half-space $N_1(X)_{K_X+\Delta<0}$.*"

page 205

• line 1, for

"For any $L \in \mathcal{L}$,"

read

"For any $V_L \in \mathcal{L}$,"

• line 6, for

"if there are infinitely rays V_L,"

read

"if there are infinitely many rays V_L in that half-space,"

• line 8, for

"*Fourth step: for any subset $\mathcal{L}'$ of $\mathcal{L}$, the cone*

$$\overline{\mathrm{NE}}(X)_{K_X+\Delta\geq 0} + \sum_{L\in\mathcal{L}'} V_L$$

is closed."

read

"*Fourth step: for any subset $\mathcal{L}'$ of $\mathcal{L}$, the cone*

$$\overline{\mathrm{NE}}(X)_{K_X+\Delta\geq 0} + \sum_{V_L\in\mathcal{L}'} V_L$$

is closed."

page 211

• line 7, for

"If E is normal, we can proceed as in 7.45. Singularities of E will be our main concern.

Since Y is normal, $c(E)$ has codimension at least 2 in Y and $E = c^{-1}(c(E))$ (1.40)."

read

"Since Y is normal, $E = c^{-1}(c(E))$ (1.40)."

• line 7, for

"If E is normal, we can proceed as in 7.45. Singularities of E will be our main concern.

Since Y is normal, $c(E)$ has codimension at least 2 in Y and $E = c^{-1}(c(E))$ (1.40)."

read

"Replacing c with its Stein factorization, we may assume that it has connected fibers. Since Y is normal, $E = c^{-1}(c(E))$ (1.40)."

• line 13, for

"the c-ampleness of $-K_X$"

read

"the c-ampleness of $-(K_X + \Delta)$"

page 212

• line -17, for "exact sequences" read "spectral sequences"

page 219

• after Exercise 8, add a new exercise:

Let X be a smooth projective variety of dimension n such that $(-K_X)^n > 0$ and $K_X \cdot C < 0$ for every curve C on X. Show that X is a Fano variety. (*Hint*: use the base-point-free theorem 7.32.)