The right adjoint of the parabolic induction

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Abstract

We extend the results of Emerton on the ordinary part functor to the category of the smooth representations over a general commutative ring R, of a general reductive p-adic group G (rational points of a reductive connected group over a local non archimedean field F of residual characteristic p). In Emerton's work, the characteristic of F is 0, R is a complete artinian local $\mathbb{Z}_p$-algebra having a finite residual field, and the representations are admissible. We show:

The smooth parabolic induction functor admits a right adjoint. The center-locally finite part of the smooth right adjoint is equal to the admissible right adjoint of the admissible parabolic induction functor when R is noetherian. The smooth and admissible parabolic induction functors are fully faithful when p is nilpotent in R.

Contents

1 Introduction

Let R be a commutative ring, let F be a local non archimedean field of finite residual field of characteristic p, let $\mathbf{G}$ be a reductive connected F-group. Let $\mathbf{P}, \overline{\mathbf{P}}$ be two opposite parabolic F-subgroups of unipotent radicals $\mathbf{N}, \overline{\mathbf{N}}$ and same Levi subgroup $\mathbf{M} = \mathbf{P} \cap \overline{\mathbf{P}}$. Let $\mathbf{A_M}$ be the maximal F-split central subtorus of $\mathbf{M}$. The groups of F-points are denoted by the same letter but not in bold. The parabolic induction functor $\mathrm{Ind}_P^G : \mathrm{Mod}_R^\infty(M) \to$

$\mathrm{Mod}_R^\infty(G)$ between the categories of smooth R-representations of M and of G, is the right adjoint of the N-coinvariant functor, and respects admissibility.

For any (R, F, G), we show that Ind_P^G admits a right adjoint R_P^G.

When R is noetherian, we show that the A_M-locally finite part of R_P^G respects admissibility, hence is the right adjoint of the functor Ind_P^G between admissible R-representations.

When 0 is the only infinitely p-divisible element in R, we show that the counit of the adjoint pair $(-_N, \mathrm{Ind}_P^G)$, is an isomorphism. Therefore, Ind_P^G is fully faithful and the unit of the adjoint pair $(\mathrm{Ind}_P^G, R_P^G)$ is an isomorphism.

The results of this paper have already be used in [HV] to compare the parabolic and compact inductions of smooth representations over an algebraically closed field R of characteristic p for any pair $(F, \mathbf{G})$, following the arguments of Herzig when the characteristic of F is 0 and $\mathbf{G}$ is split. The comparison is a basic step in the classification of the non-supersingular admissible irreducible representations of G (work in progress with Abe, Henniart, and Herzig, see also Ly's work [Ly] for $GL(n, D)$ where D/F is a finite dimensional division algebra).

When p is invertible in R, it was known that Ind_P^G has a right adjoint, called also the "second adjoint". When R is the field of complex numbers, Casselman for admissible representations and Bernstein in general proved that the right adjoint is equal to the $\overline{N}$-coinvariant functor multiplied by the modulus of P. Another proof was published by Bushnell [Bu]. Both proofs rely on the property that the category $\mathrm{Mod}_{\mathbb{C}}(G)$ is noetherian. Conversely, Dat [Dat] proved that the second adjointess implies the noetheriannity of $\mathrm{Mod}_R(G)$ and prove it assuming the existence of certain idempotents (constructed using the theory of types for linear groups, classical groups if $p \neq 2$, and groups of semi-simple rank 1). Under this hypothesis on G, Dat showed also that the N-coinvariant functor respects admissibility.

When the characteristic of F is 0 and R is a complete artinian local $\mathbb{Z}_p$-algebra having finite residual field, Emerton [Emerton] showed that Ind_P^G restricted to admissible representations has a right adjoint equal to the ordinary part functor $\mathrm{Ord}_{\overline{P}}$. Introducing the derived ordinary functors he showed also that the N-coinvariant functor respects admissibility [Emerton2, 3.6.7 Cor].

In section 2 we give precise definitions and references to the litterature on adjoint functors and on grothendieck abelian categories.

In sections 3 and 4, the existence of a right adjoint of $\mathrm{Ind}_P^G : \mathrm{Mod}_R^\infty(M) \to \mathrm{Mod}_R^\infty(G)$ is proved using that $\mathrm{Mod}_R^\infty(G)$ is a grothendieck abelian category and that Ind_P^G is an exact functor commuting with small direct sums. This method does not apply to the functor $\mathrm{Ind}_P^G : \mathrm{Mod}_R^{adm}(M) \to \mathrm{Mod}_R^{adm}(G)$ because the category of smooth admissible R-representations is not grothendieck in general. It is not even known if it is an abelian category when R is a field of characteristic p as well as F.

In section 5, we assume that p is nilpotent in R; we show the vanishing of the N-coinvariants of ind_P^{PgP} when $PgP \neq P$ and that the counit of the adjunction $(-_N, \mathrm{Ind}_P^G)$ is an isomorphism; the general arguments of section 2 imply that the unit of the adjunction $(\mathrm{Ind}_P^G, R_P^G)$ is an isomorphism and that Ind_P^G is fully faithful. When R is noetherian, $\mathrm{Ind}_P^G : \mathrm{Mod}_R^{adm}(M) \to \mathrm{Mod}_R^{adm}(G)$ is also obviously fully faithful.

In section 6, we replace G by its open dense subset $P\overline{P}$. The partial compact induction functor $\mathrm{ind}_P^{P\overline{P}} : \mathrm{Mod}_R^\infty(M) \to \mathrm{Mod}_R^\infty(\overline{P})$ admits a right adjoint $R_P^{P\overline{P}}$ by the general method of section 2. Let $\mathrm{Res}_{\overline{P}}^G : \mathrm{Mod}_R(G) \to \mathrm{Mod}_R(\overline{P})$ be the restriction functor. Let A_M be the split center of M. We fix an element $z \in A_M$ strictly contracting N. We prove that the z-locally finite parts of R_P^G and of $R_P^{P\overline{P}} \circ \mathrm{Res}_{\overline{P}}^G$ are isomorphic. The right adjoint $R_{\overline{P}}^{\overline{P}P} : \mathrm{Mod}_R^\infty(P) \to \mathrm{Mod}_R^\infty(M)$ of $\mathrm{ind}_{\overline{P}}^{\overline{P}P}$ is explicit: it is the smooth part of the functor $\mathrm{Hom}_{R[N]}(C_c^\infty(N, R), -)$.

In section 7, following Casselman and Emerton, we give the Hecke description of the above functor $R_{\overline{P}}^{\overline{P}P} : \mathrm{Mod}_R^\infty P \to \mathrm{Mod}_R^\infty(M)$. We fix an open compact subgroup N_0 of N. The submonoid M^+ of elements of M contracting N_0 acts on V^{N_0} by the Hecke action. We have the smooth induction functor $\mathrm{Ind}_{M^+}^M : \mathrm{Mod}_R^\infty(M^+) \to \mathrm{Mod}_R^\infty(M)$. We show that $R_{\overline{P}}^{\overline{P}P}$ is the functor $V \mapsto \mathrm{Ind}_{M^+}^M(V^{N_0})$. The A_M-locally finite part of this functor is the Emerton's ordinary part functor $\mathrm{Ord}_P : \mathrm{Mod}_R^\infty P \to \mathrm{Mod}_R^\infty(M)$.

In section 8 we assume that R is noetherian and we show that $\mathrm{Ord}_P(V)$ is admissible when V is an admissible R-representation of G. Therefore the parabolic induction functor $\mathrm{Ind}_P^G : \mathrm{Mod}_R^{adm} M \to \mathrm{Mod}_R^{adm} G$ admits a right adjoint equal to the functor $\mathrm{Ord}_{\overline{P}}^G : \mathrm{Ord}_{\overline{P}} \circ \mathrm{Res}_{\overline{P}}^G$. The unit of the adjunction $(\mathrm{Ind}_P^G, \mathrm{Ord}_{\overline{P}})$ is an isomorphism.

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2 Review on adjunction between grothendieck abelian categories

We fix an universe $\mathcal{U}$ and we denote by $\mathbb{S}et$ the category of $\mathcal{U}$-sets, i.e. belonging to $\mathcal{U}$. In a small category, the set of objects is $\mathcal{U}$-small, i.e. isomorphic to a $\mathcal{U}$-set, as well as the set of morphisms $\mathrm{Hom}(A,B)$ for any objects A and B. In a locally small category, only the set $\mathrm{Hom}(A,B)$ is supposed to be $\mathcal{U}$-small. (In [KS, 1.1, 1.2], small is called $\mathcal{U}$-small, and a locally small category is called a $\mathcal{U}$-category.)

Let $\mathcal{I}$ be a small category and let $\mathcal{C},\mathcal{D}$ be locally small categories. We denote by $\mathcal{C}^{op}$ the opposite category of $\mathcal{C}$ and by $\mathcal{D}^{\mathcal{C}}$ the category of functors $\mathcal{C}\to\mathcal{D}$. A contravariant functor $\mathcal{C}\to\mathcal{D}$ is a functor $\mathcal{C}^{op}\to\mathcal{D}$. The categories $\mathbb{S}et^{\mathcal{C}^{op}}, \mathbb{S}et^{\mathcal{C}}$ are not locally small in general (if $\mathcal{C}$ is not small) [KS, Def. 1.4.2].

Proposition 2.1. [KS, Def. 1.2.11, Cor. 1.4.4]
The contravariant Yoneda functor : $C \mapsto \mathrm{Hom}(C,-) : \mathcal{C} \to \mathbb{S}et^{\mathcal{C}}$ *and the covariant Yoneda functor :* $C \mapsto \mathrm{Hom}(-,C) : \mathcal{C} \to \mathbb{S}et^{\mathcal{C}^{op}}$ *are fully faithful.*

A functor F in $\mathbb{S}et^{\mathcal{C}}$ or in $\mathbb{S}et^{\mathcal{C}^{op}}$ is called representable when it is isomorphic to the image of an object $C\in\mathcal{C}$ by the Yoneda functor [KS, Def.1.4.8]. The object C which is unique modulo unique isomorphism is called a representative of F.

A functor $F:\mathcal{I}\to\mathcal{C}$ defines functors

$$\varinjlim F \in \mathbb{S}et^{\mathcal{C}} \quad C \mapsto \mathrm{Hom}_{\mathcal{C}^{\mathcal{I}}}(F, ct_C), \quad \varprojlim F \in \mathbb{S}et^{\mathcal{C}^{op}} \quad C \mapsto \mathrm{Hom}_{\mathcal{C}^{\mathcal{I}}}(ct_C, F),$$

where $ct_C : \mathcal{I}\to\mathcal{C}$ is the constant functor defined by $C\in\mathcal{C}$. When the functor $\varinjlim F$ is representable, a representative is called the injective limit (or colimit or direct limit) of F, is denoted also by $\varinjlim F$, and we have natural isomorphism [ML, III.4 (2), (3)]

$$\varinjlim F(C) = \mathrm{Hom}_{\mathcal{C}^{\mathcal{I}}}(F, ct_C) \simeq \mathrm{Hom}_{\mathcal{C}}(\varinjlim F, C).$$

When the functor $\varprojlim F$ is representable, a representative is called the projective limit (or inverse limit or limit) of F, is denoted also by $\varprojlim F$, and we have natural isomorphism

$$\varprojlim F(C) = \mathrm{Hom}_{\mathcal{C}^{\mathcal{I}}}(ct_C, F) \simeq \mathrm{Hom}_{\mathcal{C}}(C, \varprojlim F).$$

One says also that $(F(i))_{i\in\mathcal{I}}$ is an inductive or projective system in $\mathcal{C}$ indexed by $\mathcal{I}$ or $\mathcal{I}^{op}$ and one writes $\varinjlim (F(i))_{i\in\mathcal{I}}$ or $\varprojlim (F(i))_{i\in\mathcal{I}^{op}}$ for the object $\varinjlim F$ or $\varprojlim F$.

Example 2.2. 1) A set of objects $(C_i)_{i\in\mathcal{I}}$ of $\mathcal{C}$ indexed by a set $\mathcal{I}$ can be viewed as a functor $F:\mathcal{I}\to\mathcal{C}$ where $\mathcal{I}$ is identified with a discrete category (the only morphisms are the identities). When they exist, $\varinjlim F=\oplus_{i\in\mathcal{I}}C_i$ is the direct sum, or coproduct, or disjoint union $\sqcup_{i\in\mathcal{I}}C_i$, and $\varprojlim F=\prod_{i\in\mathcal{I}}C_i$ is the direct product.

2) When $\mathcal{I}$ has two objects and two parallel morphisms other than the identities, a functor $F:\mathcal{I}\to\mathcal{C}$ is nothing but two parallels arrows $C_1\underset{f}{\overset{g}{\rightrightarrows}}C_2$ in $\mathcal{C}$. When they are representable, $\varinjlim F$ is the cokernel of (f,g) and $\varprojlim F$ is its kernel [KS, Def. 2.2.2].

3) When they are representable, it is possible to construct the inductive (resp. projective) limit of a functor $F:\mathcal{I}\mapsto\mathcal{C}$, using only coproduct and cokernels (resp. products and kernels) [KS, Prop. 2.2.9]. If $\mathrm{Hom}(\mathcal{I})$ denotes the set of morphisms $s:\sigma(s)\to\tau(s)$ with $\sigma(s),\tau(s)\in\mathcal{I}$, of the category $\mathcal{I}$,

$$(1)\qquad \varinjlim F \ \text{ is the cokernel of } f,g:\oplus_{s\in\mathrm{Hom}(\mathcal{I})}F(\sigma(s))\underset{f}{\overset{g}{\rightrightarrows}}\oplus_{i\in\mathcal{I}}F(i),$$

where f,g correspond respectively to the two morphisms $\mathrm{id}_{F(\sigma(s))},F(s)$, for $s\in\mathrm{Hom}(\mathcal{I})$,

$$\varprojlim F \ \text{ is the kernel of } \prod_{i\in\mathcal{I}}F(i)\underset{f}{\overset{g}{\rightrightarrows}}\prod_{s\in\mathrm{Hom}(\mathcal{I})}F(\sigma(s)),$$

where f,g are deduced from the morphisms $\mathrm{id}_{F(\tau(s))},F(s):F(\tau(s))\times F(\sigma(s))\underset{f}{\overset{g}{\rightrightarrows}}F(\tau(s))$ for $s\in\mathrm{Hom}(\mathcal{I})$.

A non-empty category $\mathcal{C}$ is called filtrant if, for any two objects C_1,C_2 there exist morphisms $C_1\to C_3,C_2\to C_3$, and for any parallel morphisms $C_1\underset{f}{\overset{g}{\rightrightarrows}}C_2$, there exists a morphism $h:C_2\to C_3$ such that $h\circ f=h\circ g$ [KS, Def. 3.1.1].

Let $F:\mathcal{C}\mapsto\mathcal{D}$ be a functor. For $U\in\mathcal{D}$, we have the category $\mathcal{C}_U$ whose objects are the pairs (X,u) with $X\in\mathcal{C},u:F(X)\to U$ in $\mathrm{Hom}(\mathcal{D})$. We say that F is right exact if the category $\mathcal{C}_U$ is filtrant for any $U\in\mathcal{D}$, and that F is left exact if the functor $F^{op}:\mathcal{D}^{op}\to\mathcal{C}^{op}$ is right exact [KS, 3.3.1].

Proposition 2.3. *Let a functor $F:\mathcal{C}\mapsto\mathcal{D}$.*

1) When $\mathcal{C}$ admits finite projective limits, F is left exact if and only it commutes with finite projective limits. In this case, F commutes with the kernel of parallel arrows.

2) When $\mathcal{C}$ admits small projective limits, F is left exact and commutes with small direct products, if and only if F commutes with small projective limits.

3) The similar statements hold true for right exact functors, inductive limits, small direct sums, and cokernels.

Proof. 1) See [KS, Prop. 3.3.3, Cor. 3.3.4] .

2) If F preserves small projective limits, F is left exact and preserves small direct products (Example 2.2 1)). Conversely, from (1), a left exact functor which commutes wit small direct products preserves small projective limits because it commutes with the kernel of the parallel arrows.

3) Replace $\mathcal{C}$ by $\mathcal{C}^{op}$. □

Let $F:\mathcal{C}\to\mathcal{D}$ and $G:\mathcal{D}\to\mathcal{C}$ be two functors. Then (F,G) is a pair of adjoint functors, or F is the left adjoint of G, or G is the right adjoint if F, if their exists an isomorphism of bifunctors from $\mathcal{C}^{op}\times\mathcal{C}$ to $\mathbb{S}et$

$$\mathrm{Hom}_{\mathcal{D}}(F(.),.)\simeq\mathrm{Hom}_{\mathcal{C}}(.,G(.)),$$

called the adjunction isomorphism [KS, Def. 1.5.2]. The functor F determines the functor G up to unique isomorphism and G determines F up to unique isomorphism [KS, Thm. 1.5.3]. For $X \in \mathcal{C}$, the image of the identity $\mathrm{id}_{F(X)} \in \mathrm{Hom}_{\mathcal{D}}(F(X), F(X))$ by the adjunction isomorphism is a morphism $X \mapsto G \circ F(X)$. Similarly, for $Y \in \mathcal{D}$, the image of $\mathrm{id}_{G(Y)}$ is a morphism $F \circ G(Y) \to Y$. The morphisms are functorial in X and Y. The corresponding morphisms of functors are called the unit and the counit :

$$\epsilon : 1_{\mathcal{C}} \to G \circ F, \quad \eta : F \circ G \to 1_{\mathcal{D}}.$$

Proposition 2.4. *Let (F, G) be a pair of adjoint functors.*

F is fully faithful if and only if the unit $\epsilon : 1 \to G \circ F$ is an isomorphism.

G is fully faithful if and only if the counit $\eta : F \circ G \to 1$ is an isomorphism.

F and G are fully faithful if and only if F is an equivalence (fully faithful and essentially surjective [KS, Def. 1.2.11,1.3.13] *if and only if G is an equivalence. In this case F and G are quasi-inverse one to each other.*

Proof. See [KS, Prop. 1.5.6]. □

Proposition 2.5. *Let (F, G) be a pair of adjoint functors. Then F is right exact and G is left exact.*

Proof. See [KS, Prop. 3.3.6]. □

Let $\mathcal{A}$ be a locally small abelian category. A generator of $\mathcal{A}$ is an object $E \in \mathcal{A}$ such that the functor $\mathrm{Hom}(E, -) : \mathcal{A} \to \mathbb{S}et$ is faithful (i.e. any object of $\mathcal{A}$ is a quotient of a small direct sum $\oplus_i E$). If $\mathcal{A}$ admits small inductive limits, the functor between abelian categories

$$F \mapsto \varinjlim F : \mathcal{A}^{\mathcal{I}} \to \mathcal{A}$$

is additive and right exact.

Definition 2.6. [KS, Def. 8.3.24] *A locally small abelian category $\mathcal{A}$ is called grothendieck if it admits a generator, small inductive limits, and the small filtered inductive limits are exact.*

Example 2.7. *Given a ring $R \in \mathcal{U}$, the category of left R-modules in $\mathcal{U}$ is small, abelian, and grothendieck with generator R.*

Proof. See [KS, Ex. 8.3.25]. □

Proposition 2.8. *A grothendieck abelian locally small category admits small projective limits.*

Proof. See [KS, Prop. 8.3.27]. □

Proposition 2.9. *Let a functor $F : \mathcal{A} \to \mathcal{C}$ where $\mathcal{A}$ is a grothendieck abelian locally small category. The following properties are equivalent:*

1) F admits a right adjoint,

2) F commutes with small inductive limits,

3) F is right exact and commutes with small direct sums.

Proof. See [KS, Prop. 8.3.27]. □

A similar statement characterizes the existence of a left adjoint.

3 The category $\mathrm{Mod}_R^\infty(G)$

Let R be a commutative ring, let G be a secound countable locally profinite group (for instance, a parabolic subgroup of a reductive group), and let $(K_n)_{n\in\mathbb{N}}$ be a strictly decreasing sequence of pro-p-open subgroups of G, with trivial intersection, such that K_n normal in K_0 for all n.

3.1 $\mathrm{Mod}_R^\infty(G)$ is grothendieck

A R-representation V of G is a left $R[G]$-module. A vector $v \in V$ is called smooth when it is fixed by an open subgroup of G. The set of smooth vectors of V is a $R[G]$-submodule of V, equal to $V^\infty = \cup_{n\in\mathbb{N}} V^{K_n}$ where V^{K_n} is the submodule of $v \in V$ fixed by K_n. When every vector of V is smooth, V is called smooth. (The same definition applies to a locally profinite monoid (the maximal subgroup is open and locally profinite).)

Example 3.1. The module $C_c(G,R)$ of functions $f : G \to R$ with compact support is a $R[G \times G]$-module for the left and right translations. For $n \in \mathbb{N}$, the submodule $C_c(K_n\backslash G, R)$ of compactly supported functions left invariant by K_n, is a smooth representation of G for the right translation. These submodules form a strictly increasing sequence of union the smooth part $C_c^\infty(G,R)$ of $C_c(G,R)$.

We allow only the R-modules of cardinal $< c$ for some uncountable strong limit cardinal $c > |R|$, so that the R-representations of G form a set rather than a proper class (we work in the same artinian universe $\mathcal{U}_c$ [SGA4, Exposé 1, page 4]; the cardinal of $\mathrm{Hom}_{RG}(V,V')$ is $< c$ for two R-representations V, V' of G). The abelian category $\mathrm{Mod}_R(G)$ of left $R[G]$-modules is small, grothendieck of generator $R[G]$ (Ex. 2.7), and contains the abelian full subcategory $\mathrm{Mod}_R^\infty(G)$ of smooth R-representations of G.

Lemma 3.2. *$\mathrm{Mod}_R^\infty(G)$ is a grothendieck category of generator $\oplus_{n\in\mathbb{N}} C_c(K_n\backslash G, R)$.*

Proof. An arbitrary direct sum of smooth R-representations of G is smooth. The cokernel of two parallel arrows in $\mathrm{Mod}_R^\infty(G)$ is smooth hence $\mathrm{Mod}_R^\infty(G)$ admits small inductive limits (Ex. 2.2 3)). Small filtered inductive limits are exact because they are already exact in the grothendieck category $\mathrm{Mod}_R(G)$. □

For $W \in \mathrm{Mod}_R^\infty(G), V \in \mathrm{Mod}_R(G)$ we have $\mathrm{Hom}_{R[G]}(W,V) = \mathrm{Hom}_{R[G]}(W,V^\infty)$. The smoothification

$$V \mapsto V^\infty : \mathrm{Mod}_R(G) \to \mathrm{Mod}_R^\infty(G)$$

is the right adjoint of the inclusion $\mathrm{Mod}_R^\infty(G) \to \mathrm{Mod}_R(G)$, hence is left exact (Prop. 2.5). The smoothification is never right exact if G is not the trivial group [Viglivre, I.4.3] hence does not have a right adjoint (Prop. 2.5).

3.2 Admissibility and z-finiteness

Definition 3.3. *An R-representation V of G is called admissible when it is smooth and for any compact open subgroup H of G, the R-module V^H of H-fixed elements of V is finitely generated.*

When R is a noetherian ring, we consider the category $\mathrm{Mod}_R^{adm}(G)$. It may not have a generator or small inductive limits. Worse, it may be not abelian.

Example 3.4. Let R be an algebraically closed field of characteristic p and $G = \mathbf{G}(F)$ a group as in the introduction. Given an open pro-p-subgroup I of G, a non-zero smooth R-representation of G contains a non-zero vector fixed by I; the set of irreducible admissible R-representations of G (modulo isomorphism) is infinite. Therefore their direct sum is not admissible. But it is a quotient of a generator of $\mathrm{Mod}_R^{adm}(G)$, if a generator exists. If the quotient an admissible representation remains admissible, a generator cannot exist. The admissibility is preserved by quotient when the characteristic of F is zero [VigLang], but this is unknown when the characteristic of F is p.

Let H any subset of the center of G, and let $V \in \mathrm{Mod}_R(G)$.

Definition 3.5. *An element $v \in V$ is called H-finite if the R-module $R[H]v$ is contained in a finitely generated R-submodule of V.*

The subset V^{H-lf} of H-finite elements is a R-subrepresentation of V, called the H-locally finite part of V. When every element of V is H-finite, V is called H-locally finite. The category $\mathrm{Mod}_R^{H-lf}(G)$ of H-locally finite smooth R-representations of G is a full abelian subcategory of $\mathrm{Mod}_R^{\infty}(G)$. The H-locally finite functor

$$V \mapsto V^{H-lf} : \mathrm{Mod}_R^{\infty}(G) \to \mathrm{Mod}_R^{H-lf}(G) \tag{2}$$

is the right adjoint of the inclusion $\mathrm{Mod}_R^{H-lf}(G) \to \mathrm{Mod}_R^{\infty}(G)$.

Lemma 3.6. *If V is admissible, then V is H-locally finite.*

Proof. Let $v \in V$. As V is smooth, $v \in V^{K_n}$ for some $n \in \mathbb{N}$. As V is admissible, V^{K_n} is a finitely generated R-module. As H is central, V^{K_n} is H-stable. □

4 The right adjoint R_P^G of $\mathrm{Ind}_P^G : \mathrm{Mod}_R^{\infty}(M) \to \mathrm{Mod}_R^{\infty}(G)$

Let F be a local non archimedean field of finite residue field of characteristic p, let $\mathbf{G}$ be a reductive connected F-group. We fix a maximal F-split subtorus $\mathbf{S}$ of $\mathbf{G}$, and a minimal parabolic F-subgroup $\mathbf{B}$ of $\mathbf{G}$ containing $\mathbf{S}$. We suppose that $\mathbf{S}$ is not trivial. Let $\mathbf{U}$ be the unipotent radical of $\mathbf{B}$. The $\mathbf{G}$-centralizer $\mathbf{Z}$ of $\mathbf{S}$ is a Levi subgroup of $\mathbf{B}$. We choose a pair of opposite parabolic F-subgroups $\mathbf{P}, \overline{\mathbf{P}}$ of $\mathbf{G}$ with $\mathbf{P}$ containing $\mathbf{B}$, of unipotent radicals $\mathbf{N}, \overline{\mathbf{N}}$ and Levi subgroup $\mathbf{M} = \mathbf{P} \cap \overline{\mathbf{P}}$. Let $\mathbf{A_M} \subset \mathbf{S}$ be the maximal F-split central subtorus of $\mathbf{M}$. We denote by X the group of F-rational points of an algebraic group $\mathbf{X}$ over F, with the exception that we write $N_G(S)$ for the group of F-rational points of the $\mathbf{G}$-normalizer $N_{\mathbf{G}}(\mathbf{S})$ of $\mathbf{S}$. The finite Weyl group is $W_0 = N_{\mathbf{G}}(\mathbf{S})/\mathbf{Z} = N_G(S)/Z$. We fix a strictly decreasing sequence $(K_n)_{n \in \mathbb{N}}$ of pro-p-open subgroups of G with trivial intersection, such that for all n, K_n is normal in K_0 and has an Iwahori decomposition

$$K_n = \overline{N}_n M_n N_n = N_n M_n \overline{N}_n, \tag{3}$$

where $M_n := K_n \cap M, N_n := K_n \cap N, \overline{N}_n := K_n \cap \overline{N}$.

For $W \in \mathrm{Mod}_R^{\infty}(M)$, the representation $\mathrm{Ind}_P^G(W) \in \mathrm{Mod}_R^{\infty}(G)$ parabolically induced by W is the R-module of functions $f : G \to W$ such that $f(mngx) = mf(g)$ for $m \in M, n \in N, g \in G, x \in K_n$ where $n \in \mathbb{N}$ depends on f, with G acting by right translations. The smooth parabolic induction

$$\mathrm{Ind}_P^G : \mathrm{Mod}_R^{\infty}(M) \to \mathrm{Mod}_R^{\infty}(G)$$

is the right adjoint of the N-coinvariant functor [Viglivre, I.5.7 (i), I.A.3 Prop.]

$$V \mapsto V_N \ : \ \mathrm{Mod}_R^{\infty}(G) \to \mathrm{Mod}_R^{\infty}(M) \ .$$

The N-coinvariant functor $\mathrm{Mod}_R(P) \to \mathrm{Mod}_R(M)$ is the left adjoint of the inflation functor $\mathrm{Infl}_M^P : \mathrm{Mod}_R(M) \to \mathrm{Mod}_R(P)$ sending a representation of $M = P/N$ to the natural representation of P trivial on N.

Remark 4.1. The N-coinvariants of the inflation functor Infl_M^P is the identity functor of $\mathrm{Mod}_R M$ (the co-unit $-_N \circ \mathrm{Infl}_M^P \to 1$ of the adjunction $(-_N, \mathrm{Infl}_M^P)$ is an isomorphism).

Proposition 4.2. *The smooth parabolic induction functor* $\mathrm{Ind}_P^G : \mathrm{Mod}_R^\infty(M) \to \mathrm{Mod}_R^\infty(G)$ Ind_P^G *is exact, commutes with small direct sums, and admits a right adjoint*

$$R_P^G : \mathrm{Mod}_R^\infty(G) \to \mathrm{Mod}_R^\infty(M).$$

Proof. For $W \in \mathrm{Mod}_R^\infty(M)$, we write $C^\infty(P\backslash G, W)$ for the R-module of locally constant functions on the compact set $P\backslash G$ with values in W. We fix a continuous section

$$\varphi : P\backslash G \to G. \tag{4}$$

The R-linear map

$$f \mapsto f \circ \varphi : \mathrm{Ind}_P^G(W) \to C^\infty(P\backslash G, W) \tag{5}$$

is an isomorphism. We have a natural isomorphism

$$C^\infty(P\backslash G, W) \simeq C^\infty(P\backslash G, R) \otimes_R W \simeq C^\infty(P\backslash G, \mathbb{Z}) \otimes_{\mathbb{Z}} W. \tag{6}$$

The $\mathbb{Z}$-module $C^\infty(P\backslash G, \mathbb{Z})$ is free, because it is the union of the increasing sequence of the $\mathbb{Z}$-modules $L_n := C^\infty(P\backslash G/K_n, \mathbb{Z})$ for $n \in \mathbb{N}$, which are free of finite rank as well as the quotients L_n/L_{n+1}. Hence the tensor product by $C^\infty(P\backslash G, \mathbb{Z})$ is exact, and Ind_P^G is also exact.

The smooth parabolic induction commutes with small direct sums $\oplus_{i\in\mathcal{I}} W_i$ because a function $f \in C^\infty(P\backslash G, W)$ takes only finitely many values.

Applying Prop. 2.9 and Lemma 3.2, the parabolic induction admits a right adjoint. □

Remark 4.3. When p is invertible in R, Dat [Dat, between Cor. 3.7 and Prop. 3.8] showed that

$$R_P^G(V) = ([\mathrm{Hom}_{R[G]}(C_c^\infty(G, R), V)]^N)^\infty \quad (V \in \mathrm{Mod}_R^\infty(G)).$$

The modulus δ_P of P is well defined. When R is the field of complex numbers (Bernstein) or when G is a linear group, a classical group when $p \neq 2$, or of semi-simple rank 1 [Dat], we have:

$$R_P^G(V) \simeq \delta_P V_{\overline{N}}.$$

Let $g \in G$ and Q an arbitrary closed subgroup of G. The partial compact smooth parabolic induction functor

$$\mathrm{ind}_P^{PgQ} : \mathrm{Mod}_R^\infty(M) \to \mathrm{Mod}_R^\infty(Q)$$

associates to $W \in \mathrm{Mod}_R^\infty(M)$ the smooth representation $\mathrm{ind}_P^{PgQ}(W)$ of Q by right translation on the module of functions $f : PgQ \to W$ with compact support modulo left multiplication by P ($P\backslash PgQ$ is generally not closed in the compact set $P\backslash G$) such that $f(mnghx) = mf(gh)$ for $m \in M, n \in N, h \in Q, x \in K_n \cap Q$ where $n \in \mathbb{N}$ depends on f.

Remark 4.4. When $PgP = P$, the functor $\mathrm{ind}_P^P : \mathrm{Mod}_R^\infty(M) \to \mathrm{Mod}_R^\infty(P)$ is the inflation functor Infl_M^P.

Proposition 4.5. *The functor* ind_P^{PgQ} *is exact, commutes with small direct sums, and admits a right adjoint*

$$R_P^{PgQ} : \mathrm{Mod}_R^\infty(Q) \to \mathrm{Mod}_R^\infty(M).$$

Proof. Same proof as for the functor Ind_P^G (Prop. 4.2). □

Lemma 4.6. *$W \in \mathrm{Mod}_R^\infty(M)$ is admissible if and only if $\mathrm{Ind}_P^G(W) \in \mathrm{Mod}_R^\infty(G)$ is admissible.*

Proof. This is well known and follows from the decomposition [Viglivre, I.5.6, II.2.1]:

$$(\mathrm{Ind}_P^G W)^{K_n} \simeq \oplus_{PgK_n} (\mathrm{Ind}_P^{PgK_n} W)^{K_n} \simeq \oplus_{PgK_n} W^{M \cap gK_n g^{-1}} \quad (n \in \mathbb{N}, g \in G),$$

where the sum is finite and $\mathrm{Ind}_P^{PgK_n} W \subset \mathrm{Ind}_P^G W$ is the R-submodule of functions with support contained in PgK_n. □

Corollary 4.7. *When the ring is noetherian, the smooth parabolic induction restricts to a functor, called the admissible parabolic induction,*

$$\mathrm{Ind}_P^G : \mathrm{Mod}_R^{adm}(M) \to \mathrm{Mod}_R^{adm}(G).$$

We will later show that the admissible parabolic induction admits also a right adjoint.

5 Ind_P^G is fully faithful if p is nilpotent in R

We keep the notation of the preceding section. Let Φ_G be the set of roots of S in G. We write U_α for the subgroup of G associated to a root $\alpha \in \Phi_G$ (the group $U_{(\alpha)}$ in [Bo, 21.9]).

Definition 5.1. *The p-ordinary part R_{p-ord} of R is the subset of $x \in R$ which are infinitely p-divisible.*

By [Viglivre, I (2.3.1)], $R_{p-ord} = \{0\}$ if and only if there exists no Haar measure on U_α with values in R. But p is nilpotent in R if and only if $R[1/p] = \{0\}$ if and only if

$$C_c^\infty(U_\alpha, R)_{U_\alpha} = \{0\}. \tag{7}$$

When R is a field, $R_{p-ord} \neq \{0\}$ if and only if p is nilpotent in R if and only if the characteristic of R is $\neq p$.

Proposition 5.2. *We suppose that p is nilpotent in R. Let $W \in \mathrm{Mod}_R^\infty M$ and $g \in G$. The N-coinvariants of $\mathrm{ind}_P^{PgP}(W)$ is 0 if $PgP \neq P$.*

Proof. We identify $\mathrm{ind}_P^{PgP}(W)$ with $C_c^\infty(P\backslash PgP, R) \otimes_R W$ as in (5). The action of N on $C_c^\infty(P\backslash PgP, R) \otimes_R W$ is trivial on W and is the right translation on $C_c^\infty(P\backslash PgP, R)$. Therefore

$$(\mathrm{ind}_P^{PgP}(W))_N = C_c^\infty(P\backslash PgP, R)_N \otimes_R W,$$

and we can forget W. To show that $C_c^\infty(P\backslash PgP, R)_N = 0$ if $PgP \neq P$, we prove that there exists a B-positive root α such that $U_\alpha \subset N$ and the space $P\backslash PgP$ is of the form $X \times U_\alpha$ where the right action of U_α on $P\backslash PgP$ is trivial on X and equals the natural right action on U_α. Therefore

$$C_c^\infty(P\backslash PgP, R)_{U_\alpha} = C_c^\infty(X, R) \otimes_R C_c^\infty(U_\alpha, R)_{U_\alpha}.$$

Applying (7), we obtain $C_c^\infty(P\backslash PgP)_{U_\alpha} = 0$ hence $C_c^\infty(P\backslash PgP, R)_N = 0$.

It remains to explain the existence of such an α. As $(B, N_G(S))$ is a Tits system in G [BT1, 1.2.6], we have $PgP = P\nu P$ for an element $\nu \in N_G(S)$; we can suppose that the image w of ν in W_0 has minimal length in the double coset $W_{0,M}\backslash W_0/W_{0,M}$ (where $W_{0,M} := N_M(S)/Z$). This implies that the fixator $N_\nu := \{n \in N \mid P\nu n = P\nu\}$ of $P\nu$ in N is generated by the U_α for the roots $\alpha \in \Phi_G - \Phi_M$ such that α and $w(\alpha)$ are reduced, B-positive. The fixator of $P\nu$ in M is a parabolic subgroup Q and the fixator of $P\nu$ in P is QN_ν. The group N is directly spanned by the U_β ($\beta \in \Phi_G - \Phi_M$ positive and reduced) taken in any order [Bo, 21.12]. As $PgP \neq P$, i.e. $w \neq 1$, there exists a reduced positive root $\alpha \in \Phi_G - \Phi_M$ such that $U_\alpha \not\subset N_\nu$. Such an α satisfies all the properties that we want. □

Theorem 5.3. *We suppose that p is nilpotent in R. Then*

1. *The parabolic induction* $\mathrm{Ind}_P^G : \mathrm{Mod}_R^\infty(M) \to \mathrm{Mod}_R^\infty(G)$ *is fully faithful,*
2. *The unit* $\mathrm{id}_{\mathrm{Mod}_R^\infty(M)} \to R_P^G \circ \mathrm{Ind}_P^G$ *of the adjoint pair* $(\mathrm{Ind}_P^G, R_P^G)$ *is an isomorphism.*
3. *The counit* $\eta : -_N \circ \mathrm{Ind}_P^G \to \mathrm{id}_{\mathrm{Mod}_R^\infty(M)}$ *of the adjoint pair* $(-_N, \mathrm{Ind}_P^G)$ *is an isomorphism.*

Proof. By Lemma 3.2 and Prop. 2.4, the three properties are equivalent. We prove that the counit η of the adjoint pair $(-_N, \mathrm{Ind}_P^G)$ is an isomorphism.

a) It is well known that Ind_P^G admits a finite filtration $F_1 \subset \ldots \subset F_r$ of quotients ind_P^{PgP}, with last quotient ind_P^P, associated to $P\backslash G/P$.

b) Beeing a right adjoint, the N-coinvariant functor $\mathrm{Mod}_R^\infty(P) \to \mathrm{Mod}_R^\infty(M)$ is right exact.

c) Apply Prop. 5.2 and Remarks 4.1, 4.4. □

6 z-locally finite parts of R_P^G and of $R_P^{P\overline{P}} \circ \mathrm{Res}_{\overline{P}}^G$ are equal

We keep the notation of the preceding section. We fix an element $z \in A_M$ strictly contracting N : the sequence $(z^n N_0 z^{-n})_{n\in\mathbb{Z}}$ is strictly decreasing of trivial intersection and union N. We denote $N_n := z^n N_0 z^{-n}$ when $n < 0$ (N_n for $n \geq 0$ is defined in section 4).

We compare the right adjoint $R_P^G : \mathrm{Mod}_R^\infty(G) \to \mathrm{Mod}_R^\infty(M)$ of the parabolic induction Ind_P^G to the functor $R_P^{P\overline{P}} \circ \mathrm{Res}_{\overline{P}}^G$, where $\mathrm{Res}_{\overline{P}}^G : \mathrm{Mod}_R^\infty(G) \to \mathrm{Mod}_R^\infty \overline{P}$ is the restriction functor and $R_P^{P\overline{P}} : \mathrm{Mod}_R^\infty(\overline{P}) \to \mathrm{Mod}_R^\infty(M)$ is the right adjoint of the partial compact parabolic induction $\mathrm{ind}_P^{P\overline{P}}$. We denote by

$$R_P^{G,z-lf} : \mathrm{Mod}_R^\infty G \to \mathrm{Mod}_R^{z-lf} M, \quad R_P^{P\overline{P},z-lf} : \mathrm{Mod}_R^\infty \overline{P} \to \mathrm{Mod}_R^{z-lf} M,$$

the z-locally finite parts of R_P^G and of $R_P^{P\overline{P}}$.

Theorem 6.1. *The functors* $R_P^{G,z-lf}$ *and* $R_P^{P\overline{P},z-lf} \circ \mathrm{Res}_{\overline{P}}^G$ *are isomorphic.*

Proof. We want to prove that there exists an isomorphism

$$\mathrm{Hom}_{R[M]}(W, R_P^{G,z-lf}(V)) \to \mathrm{Hom}_{R[M]}(W, R_P^{P\overline{P},z-lf}(V)) \tag{8}$$

functorial in $(W,V) \in \mathrm{Mod}_R^{z-lf}(M) \times \mathrm{Mod}_R^\infty(G)$. We may replace $R_P^{G,z-lf}, R_P^{P\overline{P},z-lf}$ by $R_P^G, R_P^{P\overline{P}}$ in (8) (recall (2)). Then using the adjunctions $(\mathrm{Ind}_P^G, R_P^G)$ and $(\mathrm{ind}_P^{P\overline{P}}, R_P^{P\overline{P}})$, we reduce to find an isomorphism

$$\mathrm{Hom}_{R[G]}(\mathrm{Ind}_P^G W, V) \to \mathrm{Hom}_{R[\overline{P}]}(\mathrm{ind}_P^{P\overline{P}} W, V) \tag{9}$$

functorial in $(W,V) \in \mathrm{Mod}_R^{z-lf}(M) \times \mathrm{Mod}_R^\infty(G)$. There is an obvious functorial homomorphism because $\mathrm{ind}_P^{P\overline{P}} W$ is a submodule of $\mathrm{Ind}_P^G W$. This homomorphism, denoted by J, sends a $R[G]$-homomorphism $\mathrm{Ind}_P^G W \to V$ to its restriction to $\mathrm{ind}_P^{P\overline{P}} W$. The homomorphism J is injective because an arbitrary open subset of $P\backslash G$ is a finite disjoint union of G-translates of compact open subsets of $P\backslash P\overline{P}$ [SVZ, Prop. 5.3]. To show that J is surjective, we introduce more notations.

Let $(g, r, \overline{n}, w) \in G \times \mathbb{N} \times \overline{N} \times W$. We say that $(g, r, \overline{n}, w)$ is admissible if

$$w \in W^{M_r}, \quad P\overline{N}_r g = P\overline{N}_r \overline{n}.$$

Let $f_{r,\overline{n},w} \in \operatorname{ind}_P^{P\overline{P}}(W)$ be the function supported on $P\overline{N}_r\overline{n}$ and equal to w on $\overline{N}_r\overline{n}$. The function $gf_{r,\overline{n},w} \in \operatorname{Ind}_P^G(W)$ is supported on $P\overline{N}_r\overline{n}g^{-1}$.

We fix an element $\Phi \in \operatorname{Hom}_{R[\overline{P}]}(\operatorname{ind}_P^{P\overline{P}} W, V)$. We show that Φ belongs to the image of J if W is z-locally finite following Emerton's method [Emerton, 4.4.6, resp. 4.4.3] in two steps:

1) Φ belongs to the image of J when $\Phi(gf_{r,\overline{n},w}) = g\Phi(f_{r,\overline{n},w})$ for all admissible $(g,r,\overline{n},w)$.
2) $\Phi(gf_{r,\overline{n},w}) = g\Phi(f_{r,\overline{n},w})$ for all admissible $(g,r,\overline{n},w)$ if W is z-locally finite.

Proof of 1) Let $g_1,\dots,g_n$ in G and non-zero functions $f_1,\dots,f_n$ in $\operatorname{ind}_P^{P\overline{P}}(W)$. We show that $\sum_i g_i\Phi(f_i) = \Phi(\sum_i g_i f_i)$. We choose $r \in \mathbb{N}$ large enough, such that the f_i, viewed as elements of $C_c^\infty(\overline{N}, W)$, are left $\overline{N}_r$-invariant with values in W^{M_r}. We fix a subset X_r of G such that

$$G = \sqcup_{h\in X_r} P\overline{N}_r h, \quad P\overline{P} = \sqcup_{h\in X_r\cap\overline{N}} P\overline{N}_r h.$$

Let $Y_i \subset X_r \cap \overline{N}$ such that the support of f_i is $\sqcup_{\overline{n}\in Y_i} P\overline{N}_r\overline{n}$. For $n \in Y_i$, we have

$$f_i|_{P\overline{N}_r\overline{n}} = f_{r,\overline{n},f_i(\overline{n})}.$$

Since $G = \sqcup_{h\in X_r} P\overline{N}_r h g_i$, f_i viewed as an element of $\operatorname{ind}_P^G W$ is equal to

$$f_i = \sum_{h\in X_r} f_i|_{P\overline{N}_r h g_i}$$

where $h \in X_r$ contributes to a non zero term if and only if $P\overline{N}_r h g_i = P\overline{N}_r\overline{n}$ for some $\overline{n} \in Y_i$; when this happens $f_i|_{P\overline{N}_r h g_i} = f_{r,\overline{n},f_i(\overline{n})}$ hence $g_i\Phi(f_i|_{P\overline{N}_r h g_i}) = \Phi(g_i(f_i|_{P\overline{N}_r h g_i}))$ by the assumption of 1). We compute

$$\begin{aligned}\sum_i g_i\Phi(f_i) &= \sum_h\sum_i g_i\Phi(f_i|_{P\overline{N}_r h g_i}) = \sum_h\sum_i \Phi(g_i(f_i|_{P\overline{N}_r h g_i}))\\ &= \Phi(\sum_i g_i(\sum_h f_i|_{P\overline{N}_r h g_i})) = \Phi(\sum_i g_i f_i).\end{aligned}$$

Therefore $\sum_i g_i\Phi(f_i) = \Phi(\sum_i g_i f_i)$ for all $g_1,\dots,g_n$ in G and $f_1,\dots,f_n$ in $\operatorname{ind}_P^{P\overline{P}}(W)$, hence Φ belongs to the image of J.

Proof of 2). We assume $W \in \operatorname{Mod}_R^{z-lf}(M)$ and we prove $\Phi(gf_{r,\overline{n},w}) = g\Phi(f_{r,\overline{n},w})$. We reduce to $\overline{n} = 1$, as $f_{r,\overline{n},w} = \overline{n}^{-1} f_{r,1,w}$, $(g\overline{n}^{-1}, r, 1, w)$ is admissible, and Φ is $\overline{N}$-equivariant.

Let $(g,r,1,w)$ admissible. We may suppose $w \neq 0$. We choose $(r', r'', a) \in \mathbb{Z}\times\mathbb{N}\times\mathbb{N}$ as follows. The integer $r' \in \mathbb{Z}$ depending on (g,r), is chosen so that the projection of the compact subset $\overline{N}_r g^{-1} \subset P\overline{N}_r$ onto N via the natural homeomorphism $P\overline{N} \to N \times \overline{P}$ is contained in $N_{r'}$, i.e. $\overline{N}_r g^{-1} \subset N_{r'}\overline{P}$. The integer $r'' \in \mathbb{N}$ depending on (r,w) and on our fixed element $z \in A_M$, is chosen so that the R-submodule of $V \in \operatorname{Mod}_R^\infty(G)$, generated by $\Phi(f_{r,1,w'})$ for w' in the finitely generated R-submodule $R[z]w$, is contained in $V^{K_{r''}}$, and $r'' \geq r$. Finally, the integer $a \in \mathbb{N}$ depending on r, r'', is chosen so that $z^a N_{r'} z^{-a} \subset N_{r''} \subset N_r$.

Let $\overline{v} \in \overline{N}_r$. The set $Pz^{-a}\overline{N}_r z^a\overline{v} = P\overline{N}_r z^a\overline{v}$ is contained in $P\overline{N}_r$ as $z^{-1} \in A_M$ contracts $\overline{N}$. The restriction of $f_{r,1,w}$ to $P\overline{N}_r z^a\overline{v}$ is $f_{r,z^a\overline{v},z^a(w)}$. We deduce

$$f_{r,1,w} = \sum_{\overline{v}\in z^{-a}\overline{N}_r z^a\backslash\overline{N}_r} (z^a\overline{v})^{-1} f_{r,1,z^a(w)}.$$

We are reduced to prove $\Phi(gv^{-1}z^{-a}f_{r,1,z^a(w)}) = g\Phi(v^{-1}z^{-a}f_{r,1,z^a(w)})$. As Φ is left $\overline{P}$-equivariant, $g\Phi(v^{-1}z^{-a}f_{r,1,z^a(w)}) = gv^{-1}z^{-a}\Phi(f_{r,1,z^a(w)})$. The set $g\overline{N}_r$ is contained in

$\overline{P}N_{r'}$ and we may write $g\overline{v}^{-1}z^{-a} = \overline{p}n_{r'}z^{-a}$ with $n_{r'} \in N_{r'}, \overline{p} \in \overline{P}$. Using again that Φ is left $\overline{P}$-equivariant, we are reduced to prove

$$\Phi(n_{r'}z^{-a}f_{r,1,z^a(w)}) = n_{r'}z^{-a}\Phi(f_{r,1,z^a(w)}).$$

Applying z^a, we are reduced to prove

$$\Phi(z^a n_{r'}z^{-a}f_{r,1,z^a(w)}) = z^a n_{r'}z^{-a}\Phi(f_{r,1,z^a(w)}).$$

Let $w' \in R[z]w$ and $\overline{v} \in \overline{N}_r$. The function $f_{r,1,w'}$ viewed in $\mathrm{Ind}_P^G(W)$, of support $P\overline{N}_r$ and equal to $w' \in W^{M_r}$ on $\overline{N}_r$, is fixed by K_r. The element $\Phi(f_{r,1,w'}) \in V$ is fixed by $K_{r''}$. As $z^a N_{r'} z^{-a} \subset N_{r''} \subset N_r$, both elements $f_{r,1,z^a(w)}$ and $\Phi(f_{r,1,z^a(w)})$ are fixed by $z^a n_{r'} z^{-a}$, and the equality is obvious. □

7 The Hecke description of $R_{\overline{P}}^{\overline{P}P} : \mathrm{Mod}_R^\infty(P) \to \mathrm{Mod}_R^\infty(M)$

We keep the notation of the preceding section. The submonoid $M^+ \subset M$ contracting the open compact subgroup N_0 of N is the set of $m \in M$ such that $mN_0m^{-1} \subset N_0$; it contains the open compact subgroup M_0 of M. The union $\cup_{a\in\mathbb{N}} z^{-a}M^+$ is equal to M.

The right adjoint of the restriction functor $\mathrm{Mod}_R(M) \to \mathrm{Mod}_R(M^+)$ is the induction functor

$$I_{M^+}^M : \mathrm{Mod}_R(M^+) \to \mathrm{Mod}_R(M)$$

sending $W \in \mathrm{Mod}_R(M^+)$ to the module $I_{M^+}^M(W)$ of R-linear maps $\psi : M \to W$ such that $\psi(mx) = m\psi(x)$ for all $m \in M^+, x \in M$, where M acts by right translations. The smoothification of $I_{M^+}^M$ is the smooth induction functor

$$\mathrm{Ind}_{M^+}^M : \mathrm{Mod}_R^\infty(M^+) \to \mathrm{Mod}_R^\infty(M).$$

Definition 7.1. *Let $V \in \mathrm{Mod}_R^\infty(P)$. The monoid M^+ acts on V^{N_0} by the Hecke action $(m, v) \mapsto h_{N_0,m}(v)$,*

$$h_{N_0,m}(v) = \sum_{n\in N_0/mN_0m^{-1}} nmv \quad (m \in M^+, v \in V^{N_0}). \tag{10}$$

The Hecke action of M^+ on V^{N_0} is smooth because it extends the natural action of M_0 on V^{N_0}.

Theorem 7.2. *The functor*

$$V \mapsto \mathrm{Ind}_{M^+}^M(V^{N_0}) : \mathrm{Mod}_R^\infty(P) \to \mathrm{Mod}_R^\infty(M) \tag{11}$$

is right adjoint to the functor $\mathrm{ind}_{\overline{P}}^{\overline{P}P}$.

The theorem says that the functors $\mathrm{Ind}_{M^+}^M(-^{N_0})$ and $R_{\overline{P}}^{\overline{P}P}$ are isomorphic. Their z-locally finite parts are also isomorphic. The Emerton's ordinary functor Ord_P is the A_M-locally finite part of the functor $\mathrm{Ind}_{M^+}^M(-^{N_0})$:

$$\mathrm{Ord}_P = (\mathrm{Ind}_{M^+}^M(-^{N_0}))^{A_M-lf} : \mathrm{Mod}_R^\infty(P) \to \mathrm{Mod}_R^{A_M-lf}(M),$$

or also the functor $\mathrm{Ord}_P^G := \mathrm{Ord}_P \circ \mathrm{Res}_P^G : \mathrm{Mod}_R^\infty(G) \to \mathrm{Mod}_R^{A_M-lf}(M)$. Applying Thm. 6.1, we obtain:

Corollary 7.3. *The functor $R_{\overline{P}}^{G,z-lf}$ is isomorphic to the functor*

$$V \mapsto (\mathrm{Ind}_{M^+}^M(V^{N_0}))^{z-lf} : \mathrm{Mod}_R^\infty(G) \to \mathrm{Mod}_R^{z-lf}(M)$$

The functor $R_{\overline{P}}^{G,A_M-lf}$ is isomorphic to the Emerton's ordinary functor Ord_P^G.

To prove that $(\mathrm{ind}_{\overline{P}}^{\overline{P}P}, \mathrm{Ind}_{M^+}^M(-^{N_0}))$ is an adjoint pair, we view $\mathrm{ind}_{\overline{P}}^{\overline{P}P}$ as

$$C_c^\infty(N,R) \otimes_R - : \mathrm{Mod}_R^\infty(M) \to \mathrm{Mod}_R^\infty(P),$$

where $P = MN$ acts on $C_c^\infty(N,R)$ by:

$$mf : x \mapsto f(m^{-1}xm), \ nf : x \mapsto f(xn), \quad (m,n,f) \in M \times N \times C_c^\infty(N,R).$$

(In particular $m1_{N_0} = 1_{mN_0m^{-1}}, n1_{N_0} = 1_{N_0n^{-1}}$). The right adjoint is well known:

Lemma 7.4. *The smoothification of the functor*

$$\mathrm{Hom}_{R[N]}(C_c^\infty(N,R), -) : \mathrm{Mod}_R^\infty(P) \to \mathrm{Mod}_R(M)$$

is the right adjoint of the functor $\mathrm{ind}_{\overline{P}}^{\overline{P}P}$.

The following proposition 7.5 implies that the functors $\mathrm{Hom}_{R[N]}(C_c^\infty(N,R), -)$ and

$$I_{M^+}^M(-^{N_0}) : \mathrm{Mod}_R^\infty(P) \to \mathrm{Mod}_R(M).$$

are isomorphic.Therefore the same is true for their smoothifications, $R_{\overline{P}}^{\overline{P}P}$ and $\mathrm{ind}_{M^+}^M(-^{N_0})$, and Theorem 7.2 is proved.

Let $V \in \mathrm{Mod}_R^\infty(P)$. We check that the value at 1_{N_0}

$$f \mapsto f(1_{N_0}) : \mathrm{Hom}_{R[N]}(C_c^\infty(N,R), V) \to V^{N_0}$$

is M^+-equivariant. As usual, $p \in P$ acts on f by $pf = p \circ f \circ p^{-1}$. In particular, for $m \in M$,

$$(mf)(1_{N_0}) = mf(m^{-1}1_{N_0}) = mf(1_{m^{-1}N_0m}).$$

For $m \in M^+$, we obtain

$$\begin{aligned}(mf)(1_{N_0}) &= m \sum_{n^{-1} \in N_0 \backslash m^{-1}N_0m} f(1_{N_0n^{-1}}) = \sum_{n^{-1} \in N_0 \backslash m^{-1}N_0m} mnf(1_{N_0}) \\ &= \sum_{n \in N_0/mN_0m^{-1}} nmf(1_{N_0}) = h_{N_0,m}(f(1_{N_0})) \ .\end{aligned}$$

By the adjunction $(\mathrm{Res}_{M^+}^M, I_{M^+}^M)$, the value at 1_{N_0} induces an M-equivariant map

(12) $\quad \Phi : \mathrm{Hom}_{R[N]}(C_c^\infty(N,R), V) \to I_{M^+}^M(V^{N_0}) \quad f \mapsto \Phi(f)(m) = (mf)(1_{N_0}) \ (m \in M).$

Proposition 7.5. *The map Φ is an isomorphism of $R[M]$-modules.*

Proof. Φ is injective because the $R[P]$-module $C_c^\infty(N,R)$ is generated by 1_{N_0}. Indeed let $f \in \mathrm{Hom}_{R[N]}(C_c^\infty(N,R), V)$ such that $\Phi(f) = 0$. Then $f_\psi(m1_{N_0}) = f(1_{m^{-1}N_0m}) = 0$ for all $m \in M$. As f is N-equivariant, $0 = f((mn)^{-1}1_{N_0}) = f(1_{m^{-1}N_0mn})$ for all $n \in N$, hence $f = 0$.

Φ is surjective because for $\psi \in I_{M^+}^M(V^{N_0})$, there exists $f_\psi \in \mathrm{Hom}_{R[N]}(C_c^\infty(N,R), V)$ such that $f_\psi(m1_{N_0}) = m(\psi(m^{-1}))$ for all $m \in M$. We have $\Phi(f_\psi) = \psi$. The function f_ψ exists because, for all $a \in \mathbb{N}$,

$$z^a(\psi(z^{-a})) = z^a(\psi(zz^{-a-1})) = \sum_{n \in z^aN_0z^{-a}/z^{a+1}N_0z^{-a-1}} nz^{a+1}(\psi(z^{-a-1})).$$

(Note that the $R[N]$-module $C_c^\infty(N,R)$ is generated by $(1_{z^aN_0z^{-a}})_{a \in \mathbb{N}}$, and that the values at $1_{z^aN_0z^{-a}} = z^a1_{N_0}$ identify $\mathrm{Hom}_{R[N]}(C_c^\infty(N,R), V)$ with the set of sequences $(v_a)_{a \in \mathbb{N}}$ in V such that $v_a = \sum_{n \in z^aN_0z^{-a}/z^{a+1}N_0z^{-a-1}} nv_{a+1}$.) ∎

Remark 7.6. *For $V \in \mathrm{Mod}_R^\infty(P)$, a z^{-1}-finite element $\varphi \in I_{M^+}^M(V^{N_0})$ is smooth:*

$$(\mathrm{Ind}_{M^+}^M(V^{N_0}))^{z^{-1}-lf} = (I_{M^+}^M(V^{N_0}))^{z^{-1}-lf}.$$

Proof. By hypothesis $R[z^{-1}]\varphi$ is contained in a finitely generated R-submodule W_φ of $I_{M^+}^M(V^{N_0})$. The image of W_φ by the map $f \mapsto f(1)$ is a finitely generated R-submodule of V^{N_0} containing $\varphi(z^{-a})$ for all $a \in \mathbb{N}$. Since the Hecke action of M^+ on V^{N_0} is smooth, there exists a large integer $r \in \mathbb{N}$ such that M_r fixes $\varphi(z^{-a})$ for all $a \in \mathbb{N}$. As $M = \cup_{a\in\mathbb{N}} M^+ z^{-a}$, two elements of $I_{M^+}^M(V^{N_0})$ equal on z^{-a} for all $a \in \mathbb{N}$ are equal. Hence φ is fixed by M_r, φ is smooth. □

Remark 7.7. *Let $W \in \mathrm{Mod}_R^\infty(M^+)$ and $r \in \mathbb{N}$. An element $f \in I_{M^+}^M(W)$ is fixed by M_r if and only if $f(z^a)$ is fixed by M_r for all $a \in \mathbb{Z}$. The map*

$$f \mapsto f|_{z^{\mathbb{Z}}} : (I_{M^+}^M W)^{M_r} \to I_{z^{\mathbb{N}}}^{z^{\mathbb{Z}}}(W^{M_r})$$

is a $R[z^{\mathbb{Z}}]$-isomorphism.

Proof. This is an easy consequence of $(m_r f)(m^+ z^a) = f(m^+ z^a m_r) = f(m^+ m_r z^a) = m^+ m_r(f(z^a))$ for $(m^+, m_r, a) \in M^+ \times M_r \times \mathbb{Z}$. □

8 The right adjoint $\mathrm{Ord}_{\overline{P}}$ of $\mathrm{Ind}_P^G : \mathrm{Mod}_R^{adm}(M) \to \mathrm{Mod}_R^{adm}(G)$

We keep the notation of the preceding section. We suppose that the commutative ring R is noetherian.

Theorem 8.1. *For $V \in \mathrm{Mod}_R^{adm}(G)$, the representation $(I_{M^+}^M(V^{N_0}))^{z^{-1}-lf}$ of M is admissible.*

Proof. By Remark 7.6, the representation $(I_{M^+}^M(V^{N_0}))^{z^{-1}-lf}$ of M is smooth. Let $r \in \mathbb{N}$. Note that $M_r N_0$ is a group. By Remark 7.7, the map $f \mapsto f|_{z^{\mathbb{Z}}}$ is an $R[z^{\mathbb{Z}}]$-isomorphism from the M_r-fixed elements of $(I_{M^+}^M(V^{N_0}))^{z^{-1}-lf}$ to

$$X = (I_{z^{\mathbb{N}}}^{z^{\mathbb{Z}}}(V^{N_0 M_r}))^{z^{-1}-lf}.$$

We have $X \subset I_{z^{\mathbb{N}}}^{z^{\mathbb{Z}}}(Y)$ where Y is the image of X by $f \mapsto f(1)$, and is a $z^{\mathbb{N}}$-submodule of $V^{N_0 M_r}$ (for the Hecke action) containing $f(z^a)$ for all $a \in \mathbb{Z}$. We have the compact open subgroup $\overline{N}_r M_r N_0$. We will prove (Prop. 8.2) that

$$Y \subset V^{\overline{N}_r M_r N_0}.$$

Admitting this, Y is a finitely generated R-module because V is admissible and R is noetherian. The action $h_{N_0,z}$ of z on Y is surjective because, for $f \in X$ we have $f(1) = f(zz^{-1}) = h_{N_0,z} f(z^{-1})$. A surjective endomorphism of a finitely generated R-module is bijective (this is an application of Nakayama lemma [Matsumura, Thm. 2.4]). Hence the action of z on Y is bijective. Hence $Y \simeq I_{z^{\mathbb{N}}}^{z^{\mathbb{Z}}}(Y)$ is a finitely generated R-module. As R is noetherian, X is a finitely generated R-module. Therefore $(I_{M^+}^M(V^{N_0}))^{z^{-1}-lf}$ is admissible. □

Proposition 8.2. *If $f \in (I_{z^{\mathbb{N}}}^{z^{\mathbb{Z}}}(V^{M_r N_0}))^{z^{-1}-lf}$, then $f(1) \in V^{\overline{N}_r M_r N_0}$.*

Proof. We have

$$V^{M_rN_0} = \cup_{t\geq r} V^{\overline{N}_t M_r N_0}, \tag{13}$$

where $\overline{N}_t M_r N_0 = K_t M_r N_0 \subset G$ is a compact open subgroup as $M_r N_0 \subset K_0$ normalizes K_t, and the sequence $(\overline{N}_t M_r N_0)_{t\geq r}$ is strictly decreasing of intersection $M_r N_0$. We write $n(r,t) \in \mathbb{N}$ for the smallest integer such that $z^{-n}\overline{N}_r z^n \subset \overline{N}_t \subset \overline{N}_r$ for $n \geq n(r,t)$. The proof of the proposition is split in three steps.

1) $h_{N_0,z^n}(V^{\overline{N}_t M_r N_0})$ *is fixed by* $\overline{N}_r M_r N_0$ *when* $n \geq n(r,t)$.

Let $v \in V^{\overline{N}_t M_r N_0}$ and $n \geq n(r,t)$. The element $z^n v$ is fixed by $\overline{N}_r M_r$ as $\overline{N}_r M_r z^n \subset z^n \overline{N}_t M_r$. Let $\overline{n}_r \in \overline{N}_r$ and $(n_i)_{i\in I}$ a system of representatives of $N_0/z^n N_0 z^{-n}$. Using the Iwahori decomposition $\overline{N}_r M_r N_0 = N_0 \overline{N}_r M_r$ we write $\overline{n}_r n_i = n'_i \bar{b}_i$ with $n'_i \in N_0, \bar{b}_i \in \overline{N}_r M_r$. We compute:

$$\overline{n}_r h_{N_0,z^n}(v) = \sum_{i\in I} \overline{n}_r n_i z^n v = \sum_{i\in I} n'_i \bar{b}_i z^n v = \sum_{i\in I} n'_i z^n v. \tag{14}$$

We show that $(n'_i)_{i\in I}$ is a system of representatives of $N_0/z^n N_0 z^{-n}$, hence that $\overline{n}_r$ fixes $h_{N_0,z^n}(v)$, hence 1). We have to prove that ${n'_i}^{-1} n'_j \in z^n N_0 z^{-n}$ implies $i = j$. We write ${n'_i}^{-1} n'_j = \bar{b}_i n_i^{-1} n_j \bar{b}_j^{-1}$ and we assume that $\bar{b}_i n_i^{-1} n_j \bar{b}_j^{-1} \in z^n N_0 z^{-n}$. Then $n_i^{-1} n_j$ belongs to the group generated by $\overline{N}_r M_r$ and $z^n N_0 z^{-n}$, which is contained in the group $z^n \overline{N}_r M_r N_0 z^{-n}$. Hence $n_i^{-1} n_j \in z^n N_0 z^{-n}$. This implies $i = j$.

2) $V^{\overline{N}_t M_r N_0}$ is stable by $h_{N_0,z}$ (hence by h_{N_0,z^n} for $n \in \mathbb{N}$).

When $t = r$, this follows from 1) because $n(t,t) = 0$. This is true for any large $t = r$. Hence the intersection $V^{M_r N_0} \cap V^{\overline{N}_t M_t N_0}$ is stable by $h_{N_0,z}$. But this intersection is $V^{\overline{N}_t M_r N_0}$ because the group generated by $M_r N_0$ and $\overline{N}_t M_t N_0$ is $\overline{N}_t M_r N_0$, as M_r contains M_t and normalizes $\overline{N}_t, M_t, N_0$. Hence 2).

3) Let f be a z^{-1}-finite element of $I_{z^{\mathbb{N}}}^{z^{\mathbb{Z}}}(V^{M_r N_0})$. The R-module generated by $f(z^{-a})$ for $a \in \mathbb{N}$ is contained in a finitely generated R-submodule of $V^{M_r N_0}$. There exists $t \geq r$ such that $f(z^{-a})$ is contained in $V^{\overline{N}_t M_r N_0}$ for all $a \in \mathbb{N}$. By 2), $f \in I_{z^{\mathbb{N}}}^{z^{\mathbb{Z}}}(V^{\overline{N}_t M_r N_0})$. We have $f(1) \in \cap_{n\geq 1} h_{N_0,z^n}(V^{\overline{N}_t M_r N_0})$. By 1), $h_{N_0,z^n}(V^{\overline{N}_t M_r N_0}) \subset V^{\overline{N}_r M_r N_0}$ when $n \geq n(r,t)$. Hence $f(1) \in V^{\overline{N}_r M_r N_0}$. The proposition is proved. □

This ends the proof of Thm. 8.1. An admissible representation of M is A_M-locally finite (Lemma 3.6). By Thm. 8.1, Remark 7.6, and Corollary 7.3, we deduce :

Corollary 8.3. *The (admissible) parabolic induction* $\mathrm{Ind}_P^G : \mathrm{Mod}_R^{adm}(M) \to \mathrm{Mod}_R^{adm}(G)$ *admits a right adjoint, equal to*

$$(R_{\overline{P}}^G)^{A_M-lf} \simeq \mathrm{Ord}_{\overline{P}}^G : \mathrm{Mod}_R^{adm}(G) \to \mathrm{Mod}_R^{adm}(M).$$

Corollary 8.4. *When p is nilpotent in R, the admissible parabolic induction* Ind_P^G *is fully faithful, and the unit* $\mathrm{id} \mapsto \mathrm{Ord}_{\overline{P}}^G \circ \mathrm{Ind}_P^G$ *of the adjunction* $(\mathrm{Ind}_P^G, \mathrm{Ord}_{\overline{P}}^G)$ *is an isomorphism.*

Proof. Lemma 4.6, Cor. 5.3. □

It is not known if the N-coinvariant functor respects admissibility when the characteristic of F is p. When R is a field where p is invertible, the N-coinvariant functor respects admissibility. For the convenience of the reader, we give the proof which is a variant of the proof of [Viglivre, II.3.4].

(i) Let R be a commutative ring (we do not assume that R is noetherian) and $V \in \mathrm{Mod}_R^\infty(G)$. For $v \in V^{N_0}$ and $a \in \mathbb{N}$, we have $h_{N_0,z^a}(v) = \sum_{n\in N_0/z^aN_0z^{-a}} nz^a v = z^a \sum_{n\in z^{-a}N_0z^a/N_0} nv$. Applying the map $\kappa : V \to V_N$, we get

$$\kappa(h_{N_0,z^a}(v)) = [N_0 : z^a N_0 z^{-a}] z^a \kappa(v). \tag{15}$$

The index $[N_0 : z^a N_0 z^{-a}]$ is a power of p which goes to infinity with a. (Note that when a power of p vanishes in R, $\kappa(h_{N_0,z^a}(v)) = 0$ when a is large.) For $r \in \mathbb{N}$ we have $\kappa(V^{M_rN_0}) \subset (V_N)^{M_r}$ because $m\kappa(v) = \kappa(mv)$ for $m \in M, v \in V$.

(ii) We assume now that p is invertible in R. The above inclusion for $r \in \mathbb{N}$ is an equality

$$\kappa(V^{M_rN_0}) = (V_N)^{M_r}. \tag{16}$$

Indeed, let $w \in (V_N)^{M_r}$ and $v \in V$ with $\kappa(v) = w$. The fixator H_r of v in the pro-p-group M_rN_0 is open of index a power of p. The element $[M_rN_0 : H_r]^{-1} \sum_{b\in M_rN_0/H_r} bv$ is well defined, is fixed by M_rN_0 and has image w in V_N. Hence (16). As V_N is a smooth representation of M and $V^{N_0} = \cup_{r\in\mathbb{N}} V^{M_rN_0}$, (16) implies $\kappa(V^{N_0}) = V_N$ and by (13),

$$\cup_{t\geq r}\, \kappa(V^{\overline{N}_tM_rN_0}) = (V_N)^{M_r}. \tag{17}$$

Assume $a \geq n(r,t)$, by (15) and by the proof of Prop. 8.2,

$$z^a\kappa(V^{\overline{N}_tM_rN_0}) = \kappa(h_{N_0,z^a}(V^{\overline{N}_tM_rN_0})) \subset \kappa(V^{\overline{N}_rM_rN_0}). \tag{18}$$

If X is a finitely generated R-submodule of $V_N^{M_r}$, there exists $t \in \mathbb{N}$ such that $X \subset \kappa(V^{\overline{N}_tM_rN_0})$, hence by (18) there exists $a \in \mathbb{N}$ such that

$$z^a X \subset \kappa(V^{\overline{N}_rM_rN_0}). \tag{19}$$

(iii) We assume now that R is a field where p is invertible and $V \in \mathrm{Mod}_R^{adm}(G)$. By (19) the dimensions of the finite dimensional subspaces of $V_N^{M_r}$ are bounded, hence $V_N^{M_r}$ is finite dimensional. This is true for all $r \in \mathbb{N}$ therefore $V_N \in \mathrm{Mod}_R^{adm}(M)$.

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