Extensions between irreducible representations of a p-adic $GL(n)$

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Let H be the group of points of a connected reductive group over a local non archimedean field F. Let ω be a character of the center of H. Let $\mathcal{C} := \mathrm{Mod}_\omega H$ be the category of complex representations of H which are smooth (the stabilizer of a vector is an open subgroup of H), with central character ω. It is known that $\mathcal{C}$ has enough injectives and projectives, and we can define $\mathrm{Ext}^i_{\mathcal{C}}(V, V')$ for two representations $V, V' \in \mathcal{C}$, using a projective resolution $(P^i)_{i\geq 0}$ of V, or an injective resolution $(I^i)_{i\geq 0}$ of V'. The cohomology of the complex $\mathrm{Hom}_{\mathcal{C}}(P^i, V')$ and of the complex $\mathrm{Hom}_{\mathcal{C}}(V, I^i)$ are the same, and are equal to $\mathrm{Ext}^i_{\mathcal{C}}(V, V')$ by definition.

Question *Let $V, V' \in \mathcal{C}$ irreducible, with V essentially square integrable (essentially because of the center), and V' essentially tempered. Is is true that*

$$\mathrm{Ext}^i_{\mathcal{C}}(V, V') = \mathrm{Ext}^i_{\mathcal{C}}(V', V) = 0$$

for all integers $i > 0$?

This question is motivated by the orthogonal decomposition of the Schwartz algebra of H given by the Plancherel formula ([Sil th.3 page 4679] for example). I tried to prove without success that the answer was yes, some years ago while writing [Vig1]. The answer (yes) is an exercise for $GL(n, F)$ for any integer $n > 1$. It can be worth to publish it.

Let $H = G := GL(n, F)$. Let $V \in \mathcal{C}$ irreducible essentially square integrable. We can describe all the irreducible $V' \in \mathcal{C}$ such that $\mathrm{Ext}^i_{\mathcal{C}}(V', V) \neq 0$ for at least one integer $i \geq 0$. For such a V', there is a unique i such that $\mathrm{Ext}^i_{\mathcal{C}}(V', V) \simeq \mathbf{C}$, and is zero otherwise. If $V' \not\simeq V$, then V' does not have a Whittaker model. An irreducible essentially tempered representation has a Whittaker model. For all irreducible tempered representation V' not isomorphic to V, we get $\mathrm{Ext}^*_{\mathcal{C}}(V', V) = 0$. Using duality, we get $\mathrm{Ext}^*_{\mathcal{C}}(V, V') = 0$.

The computation of $\mathrm{Ext}^*_{\mathcal{C}}(V', V)$ for V irreducible essentially square integrable and V' irreducible, is a corollary of the classification of square integrable representations by Zelevinski, the theory of simple types by Bushnell and Kutzko, the Zelevinski involution by Aubert, Schneider and Stuhler, the computation of $\mathrm{Ext}^*_{\mathcal{C}}(1, V')$ by Casselman.

We give a very short proof of $\mathrm{Ext}^*_{\mathcal{C}}(V, V') = 0$ for $V,\ V' \in \mathcal{C}$, irreducible tempered and not isomorphic, suggested by Waldspurger. The group G has the particularity to have at most one irreducible tempered representation with a given infinitesimal character (i.e. cuspidal support), and $\mathrm{Ext}^*_{\mathcal{C}}(V, V') = 0$ for two irreducible representations V, V' of G having different infinitesimal characters. This second fact is very general, and uses the interpretation by Yoneda of $\mathrm{Ext}^n_{\mathcal{C}}(V, V')$ by n-extensions, as in the real case.

The author was supported by a grant of the von Humboldt foundation, and this work was done in the wonderful atmosphere of the Max Planck Institute in Bonn in the fall of 1996.

1 We set $G := GL(n, F)$ and $\mathcal{C} = \mathrm{Mod}\, G$ (we do not fix the central character). From Bernstein [Z 9.3], any $V \in \mathcal{C}$ irreducible essentially square integrable is a *Steinberg representation* $St_k(\rho)$ where ρ is an irreducible cuspidal representation of $GL(r, F)$ for some integer $r > 0$, and $rk = n$. The Steinberg representation $St_k(\rho)$ is the unique irreducible subquotient with a Whittaker model in the natural representation of G in the space of locally constant functions $f : G \to \otimes^k \rho$ such that $f(mug) = \otimes^k \rho(m) f(g)$ for any $g \in G$ and any element mu $(m \in M,\ u \in U)$, in a parabolic subgroup of G with Levi component M isomorphic to $GL(r, F)^k$, and unipotent radical U. When $r = 1$ and $\rho = 1$ is the trivial character 1 of F^*, $St_n(1) = St$ is the usual Steinberg representation.

A *block* in the abelian category $\mathcal{C}$ is an indecomposable abelian subcategory which is a direct factor. There are no non trivial homomorphisms between two different blocks. The blocks are classified by the semi-simple types of Bushnell-Kutzko [BK2, BK3], and also by the irreducible cuspidal representations of Levi subgroups modulo G-conjugation, and twist by unramified characters [BD].

The *semi-simple type* of a block is a distinguished irreducible representation σ of a distinguished open compact subgroup K of G, such that the functor

$$F_\sigma : V \to \mathrm{Hom}_G(\mathrm{ind}_{G,K}\,\sigma, V)$$

is *an equivalence of categories between the block and the category of right* $\mathrm{End}_G\,\mathrm{ind}_{G,K}\,\sigma$*-modules.*

Let I be an Iwahori subgroup (unique modulo G-conjugation). A representation $V \in \mathcal{C}$ generated by the I-invariant vectors V^I, is called *unipotent*. The unipotent representations form a block, of semisimple type the trivial representation of I. Set $F_I = \mathrm{Hom}_G(\mathrm{ind}_{G,I}\,1, -)$.

Let (e,f,d), $efd = r$, be the *invariants* of ρ [Vig1 III.5]. Let q be the order of the residual field of F. Let F' be any local non archimedean field, with residual field of order $q' = q^{fd}$. We set $G' = GL(k, F')$ and $\mathcal{C}' = \mathrm{Mod}\,G'$. Denote I' an Iwahori subgroup of G'.

Bushnell and Kutzko [BK1 7.6.18] have shown that there is a natural algebra isomorphism [BK1 7.6.18, 7.6.21]

$$i : \mathrm{End}_{G'}\,\mathrm{ind}_{G',I'}\,1 \to \mathrm{End}_G\,\mathrm{ind}_{G,K}\,\sigma.$$

We get a functor Φ which is an equivalence of categories, from the *unipotent block* in $\mathcal{C}'$ to the block in $\mathcal{C}$ containing $St_k(\rho)$ such that

$$i^* \circ F_\sigma \circ \Phi' = \mathrm{Hom}_{G'}(\mathrm{ind}_{G',I'}\,1, -).$$

For any Levi subgroup M' of G', there is a similar functor Φ' which is an equivalence from the unipotent block of M' to a block in a Levi subgroup M of G. This is compatible with the normalized parabolic induction $i_{G',M'}$ and $i_{G,M}$, or restriction $r_{M',G'}$ and $r_{M,G}$, along $Q' = M'Q'_o$ and $Q = MQ_o$, where Q'_o and Q_o are suitable Borel subgroups of G' and G:

$$\Phi \circ i_{G',M'} = i_{G,M} \circ \Phi', \quad \Phi' \circ r_{M',G'} = r_{M,G} \circ \Phi$$

This is a consequence of [BK1 7.6.21].

Proposition *The functor Φ sends an essentially square integrable (resp. unitary, having a Whittaker model, essentially tempered) irreducible unipotent representation of G' to an essentially square integrable (resp. unitary, having a Whittaker model, essentially tempered) irreducible representation of G.*

For essentially square integrable see [BK1 7.7]. For unitary see [BK1 7.6.25]. The irreducible representations of G with a Whittaker model are induced from essentially square integrable representations of Levi subgroups [Z 9.11]. The assertion for the Whittaker model follows from this and the compatibility of Φ', Φ with the induction. The tempered irreducible representations of G are induced from square integrable representations [Sil 4.5.11]. Hence the assertion for essentially tempered representations.

2 We want to prove a vanishing result for Ext^1, between characters of affine Hecke algebras, directly and in an elementary way. In fact, the best method to compute Ext^* between modules for affine Hecke algebras, is to use the dictionnary with representations. This paragraph could be skipped.

The Hecke algebra $\mathrm{End}_G\,\mathrm{ind}_{G,I}\,1$ is naturally isomorphic to the *affine Hecke algebra $H_{\mathbf{C}}(n,q)$ of type A_{n-1} and parameter q* [BK1 5.6.6].

The Hecke $\mathbf{C}$-algebra $H^o_{\mathbf{C}}(n,x)$ of type A_{n-1} with parameter $x \in \mathbf{C}^*$, $x \neq 0, 1$, is the $\mathbf{C}$-algebra generated by $(s_1, \ldots, s_{n-1})$ with the relations

$(s_i + 1)(s_i - x) = 0$ $(1 \leq i \leq n-1)$,

$s_i s_j = s_j s_i$ $(1 \leq i, j \leq n-1,\ |j - i| \neq 1)$

$s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}$ $(1 \leq i \leq n-2)$.

The affine Hecke $\mathbf{C}$-algebra $H_{\mathbf{C}}(n,x)$ of type A_{n-1} with parameter x is generated by $H^o_{\mathbf{C}}(n,x)$ and t with

$tt^{-1} = t^{-1}t = 1,\ ts_i = s_{i-1}t\ (1 < i < n),\ t^2 s_1 = s_{n-1}t^2$.
Note that this description [BK 5.4 page 177] is not the Bernstein description.

The finite algebra $H^o_{\mathbf{C}}(n,x)$ is isomorphic to the group algebra $\mathbf{C}[S_n]$ of the symmetric group S_n and has two characters. For the character sign, the image of all the s_i is -1. For the trivial character, the image of all the s_i is x. The two characters extend to characters of $H_{\mathbf{C}}(n,x)$, the image of t beeing an arbitrary not zero complex element. The $H_{\mathbf{C}}(k,q)$-module $F(St)$ is a sign character of $H_{\mathbf{C}}(n,q)$.

The center of G is naturally identified with F^* diagonally enbedded in G. The center of $H_{\mathbf{C}}(n,x)$ contains t^n. The central character of St is trivial. The category of unipotent representations of G with *trivial central character* is isomorphic by the functor F_I defined in (1) to the category $\mathrm{Mod}\, H_{\mathbf{C}}(n,q)_1$ of right modules of the quotient $H_{\mathbf{C}}(n,q)_1$ of $H_{\mathbf{C}}(n,q)$ by the two-sided ideal generated by $t^n - 1$ [Vig2 I.3.14].

2.1 Lemma *Let $\chi, \chi' \in \mathcal{C} := \mathrm{Mod}\, H_{\mathbf{C}}(n,q)_1$ two characters. Then* $\mathrm{Ext}^1_{\mathcal{C}}(\chi, \chi') = 0$.

Indeed the algebras $H^o_{\mathbf{C}}(n,q)$ and $\mathbf{C}[t], t^n = 1$, are semisimple (but the quotient $H_{\mathbf{C}}(n,q)_1$ is not semisimple). If $V \in \mathcal{C}$ is an extension of χ by χ, then $hv = \chi(h)v$ for all $h \in H^o_{\mathbf{C}}(n,q),\ v \in V$, and t acting semisimply, $V \simeq \chi \oplus \chi$.

There is another proof when $n = 2$ in [DPrasad p.175 proof of the lemma 7]. Note that if we were not fixing the center, we could have extensions. I do not know how to compute directly Ext^i when $i > 1$.

When V is an extension of two different characters $\chi' \neq \chi$ in $\mathrm{Mod}\, H_{\mathbf{C}}(n,q)_1$ or in $\mathrm{Mod}\, H_{\mathbf{C}}(n,q)$, one sees that $V \simeq \chi \oplus \chi'$ by restriction to the commuting algebras $H^o_{\mathbf{C}}(n,q)$ and $\mathbf{C}[t]$.

2.2 From (2.1), $\mathrm{Ext}^1(St, St) = 0$ in $\mathrm{Mod}_1\, G$. Any irreducible square integrable unipotent representation V of G is the twist $\mathrm{St} \otimes \chi_x$ of St by an unramified character of G

$$\chi_x(g) = x^{\mathrm{val}\det g}, \quad g \in G,$$

for some $x \in \mathbf{C}^*$, where $\mathrm{val} : (F)^* \to \mathbf{Z}$ is the valuation of F, sending an uniformizing parameter to 1. The central character of $\mathrm{St} \otimes \chi_x$ is the character χ_{kx} of F^*. It is trivial if and only if $\mathrm{St} \otimes \chi_x \simeq \mathrm{St}$.

The twist by a character χ of G does not change the value of Ext^*. If $V, V' \in \mathcal{C} := \mathrm{Mod}_\omega\, G$, then $V \otimes \chi, V' \otimes \chi \in \mathcal{C}_\chi := \mathrm{Mod}_{\omega\omega(\chi)}\, G$ where $\omega(\chi)$ is the restriction of χ to the center of G. We have :

$$\mathrm{Ext}^*_{\mathcal{C}}(V, V') \simeq \mathrm{Ext}^*_{\mathcal{C}_\chi}(V \otimes \chi, V' \otimes \chi).$$

Using the functor F_I of (1), we get :

Proposition *Let V, V' irreducible and essentially square integrable in the category $\mathcal{C} := \mathrm{Mod}_\omega\, G$. Then*

$$\mathrm{Ext}^1_{\mathcal{C}}(V, V') = 0.$$

There is another proof due to Silberger of this result, valid for a general reductive group [Sil2]. To compute some $\mathrm{Ext}^i_{\mathcal{C}}(V, V')$ when $i > 1$, we use the results of Casselman [Cas].

3 Let H as in the introduction. Let $\mathcal{C} := \mathrm{Mod}_1\, H$ be the category of representations of H with trivial character. Denote by $St_Q \in \mathcal{C}$ the *Steinberg representation* defined by a parabolic subgroup Q of H [BW 4.6 page 308]. If τ_Q is the natural representation of H on the complex space of locally constant left Q-invariant functions $H \to \mathbf{C}$, then St_Q is the quotient of τ_Q by the subrepresentation generated by the natural images of $\tau_{Q'}$ in τ_Q, for all parabolic subgroups Q' of H which contain Q. We have $St_H = 1$. When $Q = Q_o$ is minimal, then $St_{Q_o} = St$ is the usual Steinberg representation. The representations St_Q are irreducible and not isomorphic.

The *parabolic rank* of Q is the rank of a maximal split torus in the center of a Levi component of Q. We denote

$$m_Q = \text{ parabolic rank of } Q \ - \ \text{parabolic rank of } H\,.$$

This an integer ≥ 0.

Theorem [BW 5.1 th.4.12 page 313] *Let* $V \in \mathcal{C} := \mathrm{Mod}_1 H$ *irreducible such that* $\mathrm{Ext}^*(1, V) \neq 0$. *Then there exists a parabolic subgroup* Q *of* H *such that* $V \simeq St_Q$. *Moreover* $\mathrm{Ext}^m_{\mathcal{C}}(1, St_Q) \simeq \mathbf{C}$ *if* $m = m_Q$, *and is zero otherwise.*

Remarque Suppose $H = G := GL(n, F)$, and $\mathcal{C} := \mathrm{Mod}_1 G$.
We have $\mathrm{Ext}^o_{\mathcal{C}}(1, 1) \simeq \mathbf{C}$ and $\mathrm{Ext}^m_{\mathcal{C}}(1, 1) = 0$ for any integer $m \geq 1$.
The representation $\tau_Q \in \mathcal{C}$ has a unique irreducible subquotient with a Whittaker model, this unique subquotient is isomorphic to St [Z 9.7]. In particular, when $Q \neq Q_o$ the representation St_Q does not have a Whittaker model. Hence $\mathrm{Ext}^*_{\mathcal{C}}(1, V) = 0$ for any irreducible representation $V \neq \mathrm{St}$ with a Whittaker model.

4 Zelevinski involution Let G as in (1). The Zelevinski involution τ in $\mathrm{Mod}\, G$ has the following properties :

a) τ respects the property of beeing irreducible [A 2.3, 2.9].
b) τ exchanges the trivial and the usual Steinberg representation [Z 9.2].
c) $\tau(- \otimes \chi) = \tau(-) \otimes \chi$ commutes with the twist by a character χ of G [Z 9.1].
d) τ respects the cuspidal support [Z 9.1].
e) τ is an exact contravariant functor and respects the cuspidal support [SS 3.1], hence respects the representations with a given central character.

Set $\mathcal{C} := \mathrm{Mod}\, G$ or $\mathcal{C} := \mathrm{Mod}_\omega G$, where ω is a character of the center of G. By e) we have for any $V, V' \in \mathcal{C}$

$$\mathrm{Ext}^*_{\mathcal{C}}(V, V') \simeq \mathrm{Ext}^*_{\mathcal{C}}(\tau(V'), \tau(V)).$$

With the notations of (3), the representation $\tau(St_Q)$ is not isomorphic to St when $Q \neq G$ by b), and is a subquotient of τ_{Q_o} by d). Hence $\tau(St_Q)$ *does not have a Whittaker model when* $Q \neq G$, in particular is not essentially tempered. We deduce from (3):

Theorem *Let* $V, V' \in \mathcal{C} := \mathrm{Mod}_w G$, *irreducible, such that* $V \simeq St \otimes \chi$ *is unipotent and essentially square integrable as in 2), and* $\mathrm{Ext}^*_{\mathcal{C}}(V', V) \neq 0$. *Then there exists a parabolic subgroup* Q *of* G' *such that* $V' \simeq \tau(St'_Q) \otimes \chi$. *For* $V' = \tau(St_Q) \otimes \chi$, *we have* $\mathrm{Ext}^m_{\mathcal{C}}(V', V) \simeq \mathbf{C}^*$ *if* $m = m_Q$ *as in 3), and zero otherwise.*

In particular, if V is a unipotent Steinberg representation, and if $V' \not\simeq V$ is essentially tempered, then

$$\mathrm{Ext}^o_{\mathcal{C}}(V, V) \simeq \mathbf{C}, \quad \mathrm{Ext}^i_{\mathcal{C}}(V, V) = \mathrm{Ext}^i_{\mathcal{C}}(V', V) = 0$$

for all integers $i > 0$. We will prove also

$$\mathrm{Ext}^i_{\mathcal{C}}(V, V') = 0 \tag{4.1}$$

using duality as follows.

5 Duality Let (H, ω) as in the introduction. The contragredient $V \to V^*$ is a contravariant exact functor in $\mathrm{Mod}\, H$, which sends a projective representation to an injective representation [Vig2 I.4.18]. A representation V is called *admissible* when $V^{**} \simeq V$. When V is admissible, and $(P_i) \to V$ is a projective resolution of V, then $V^* \to (P_i^*)$ is an injective resolution of V^*, and $\mathrm{Hom}(P_i, W) \simeq \mathrm{Hom}(W^*, P_i^*)$ canonically [Vig2 I4.13]. If $V \in \mathrm{Mod}_\omega H$, then $V^* \in \mathrm{Mod}_{\omega^{-1}} H$. Set $\mathcal{C} := \mathcal{C}^* := \mathrm{Mod}\, H$ or $\mathcal{C} := \mathrm{Mod}_\omega H$, $\mathcal{C}^* := \mathrm{Mod}_{\omega^{-1}} H$.

Proposition *Let* $V, W \in \mathcal{C}$ *admissible of contragredient* $V^*, W^* \in \mathcal{C}^*$, *one has* $\mathrm{Ext}^*_{\mathcal{C}}(V, W) \simeq \mathrm{Ext}^*_{\mathcal{C}^*}(W^*, V^*)$.

The contragredient respects the property of beeing essentially square integrable and of beeing essentially tempered. We deduce (4.1). Hence the answer to the question in the introduction is yes, for $G = GL(n, F)$.

There is another proof, suggested by Waldspurger, using that the essentially tempered irreducible representations of G have different cuspidal support. This comes from the classification of Zelevinki [Z], which shows that tempered irreducible representations are not degenerate (1), and that not degenerate irreducible representations have different cuspidal support.

6 Let $(H, w), \mathcal{C}$ as in (5). There is a natural equivalence between the two bifunctors on $\mathcal{C}$,

$$\mathrm{Ext}^n_{\mathcal{C}}(A, B) \quad \text{and} \quad \mathrm{Yext}^n_{\mathcal{C}}(A, B)$$

given by the Yoneda n-extensions of A by B modulo an equivalence relation $\equiv$. The proofs are the same than in the category of (left) modules for a ring [M III.6.4, III.8.2].

An n-extension X of A by B is an exact sequence starting at B and ending at A,

$$X \ : \ 0 \to B \to X_n \to \ldots \to X_1 \to A \to 0.$$

A morphism $\gamma : X \to Y$ between two n-extensions starting with β and ending with α is a commutative diagram

$$\begin{array}{ccccccccccccc} X: & 0 & \to & B & \to & X_n & \to & \ldots & \to & X_1 & \to & A & \to & 0 \\ \downarrow\gamma & & & \downarrow\beta & & \downarrow & & & & \downarrow & & \downarrow\alpha & & \\ Y: & 0 & \to & D & \to & Y_n & \to & \ldots & \to & Y_1 & \to & C & \to & 0 \end{array}$$

The equivalence relation $\equiv$ in the set of n-extensions of A by B, is generated by the relation : there exists a morphism $\gamma : X \to Y$ starting and ending with the identity.

An n-extension X ending at A can be spliced with an m-extension Y starting at A, to give an $n+m$-extension $X \circ Y$ starting like X, ending like Y. If $\alpha : A' \to A$, one defines by pull-back an extension $X\alpha$ starting like X, ending at A'. If Z is an m-extension starting by A', one defines by push-out an m-extension αZ starting at A, ending like Z. By definition of the equivalence relation, one has

$$X\alpha \circ Z \equiv X \circ \alpha Z.$$

A morphism $\gamma : X \to Y$ starting with β and ending with α gives an equivalence [M III.5.1]

$$\beta X \equiv Y\alpha.$$

An element z of the center of $\mathcal{C}$ defines an endomorphism of X. If z acts on A and on B by multiplication by two different scalars $z_a \neq z_B \in R$, we deduce that the image of X in $\mathrm{Yext}^n(A, B) \simeq \mathrm{Ext}^n(A, B)$ is 0.

For $A, B \in \mathcal{C}$ irreducible of different cuspidal support, there is an element z in the center of $\mathcal{C}$ which acts by the identity on A and is zero on B'. This comes from the description of the center by Bernstein [BD]. We get the following theorem.

6.1 Theorem *Let $V, V' \in \mathcal{C}$ irreducible of different cuspidal support. Then* $\mathrm{Ext}^*_{\mathcal{C}}(V, V') = 0$.

6.2 Corollary *Suppose that $H = GL(n, F)$. Let $V, V' \in \mathcal{C}$ irreducible not degenerate, and $V \neq V'$. Then* $\mathrm{Ext}^*_{\mathcal{C}}(V, V') = 0$.

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